LIQUIDITY IN CREDIT NETWORKS

A Little Trust Goes a Long Way

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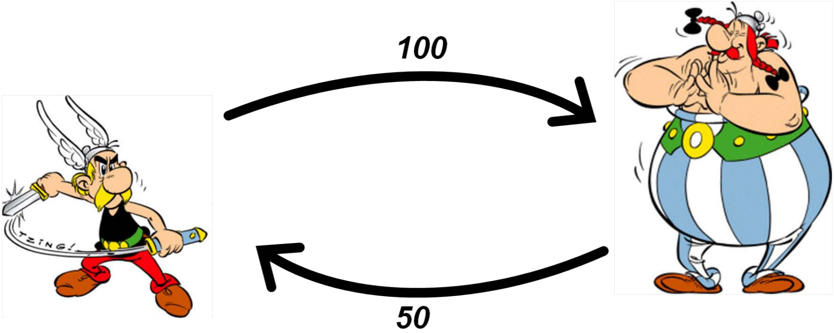
1 INTRODUCTION
- Illustrative Example
- What is a Credit Network?
- Applications

2 LIQUIDITY MODEL & ANALYSIS
- Liquidity Model
- Main Results
- Analysis

3 SIMULATIONS
- Setup
- Results

4 SUMMARY

Barter Economy in *Armorica*



Asterix willing to accept up to 100 IOUs from Obelix
Obelix willing to accept up to 50 IOUs from Asterix

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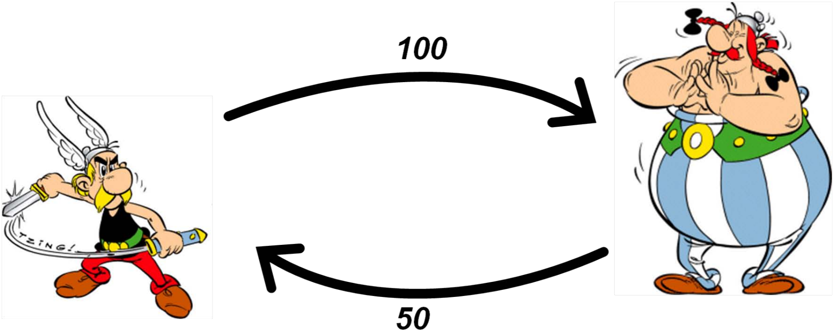

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Barter Economy in *Armorica*

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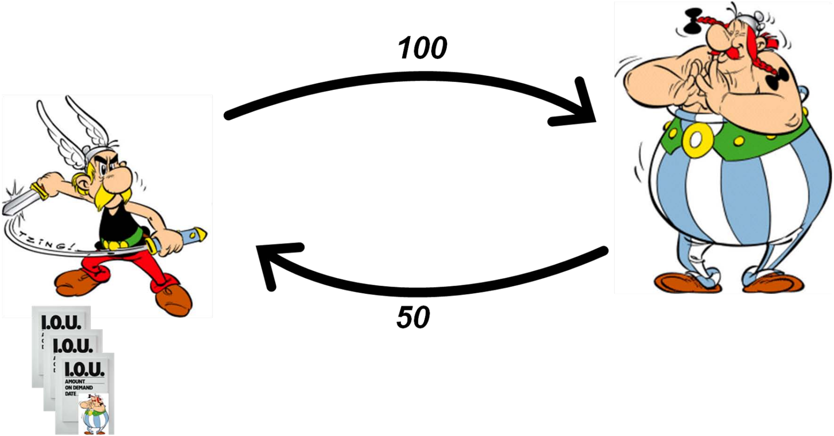

Gives Asterix 10 IOUs as payment.

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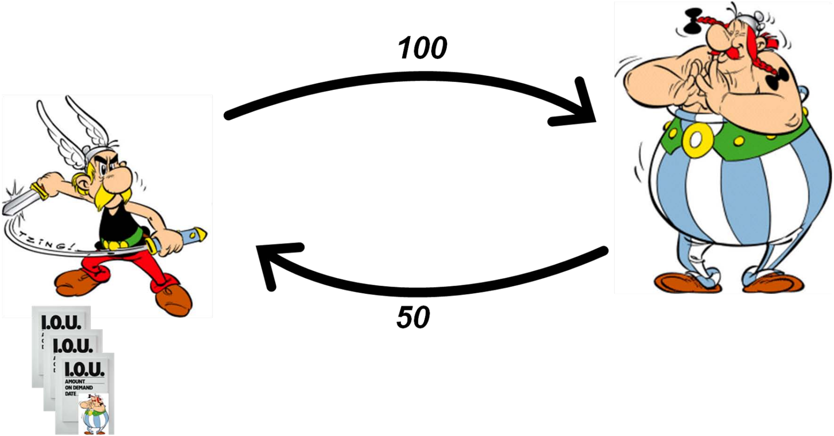

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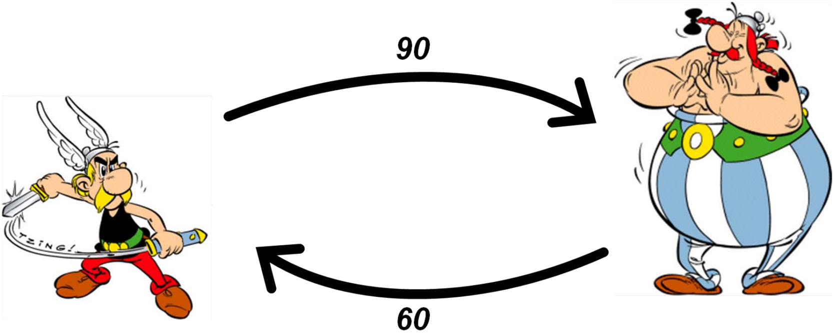
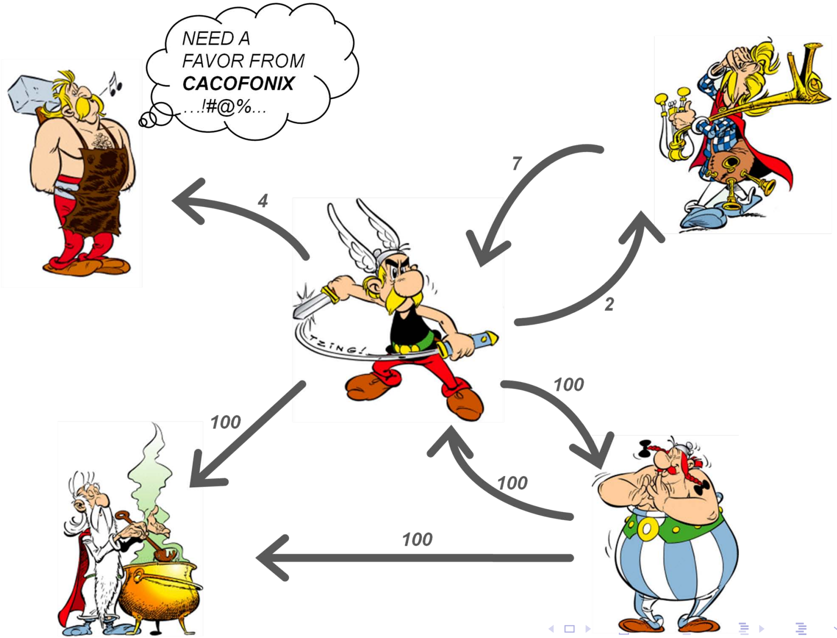
New trust values....




NEED A
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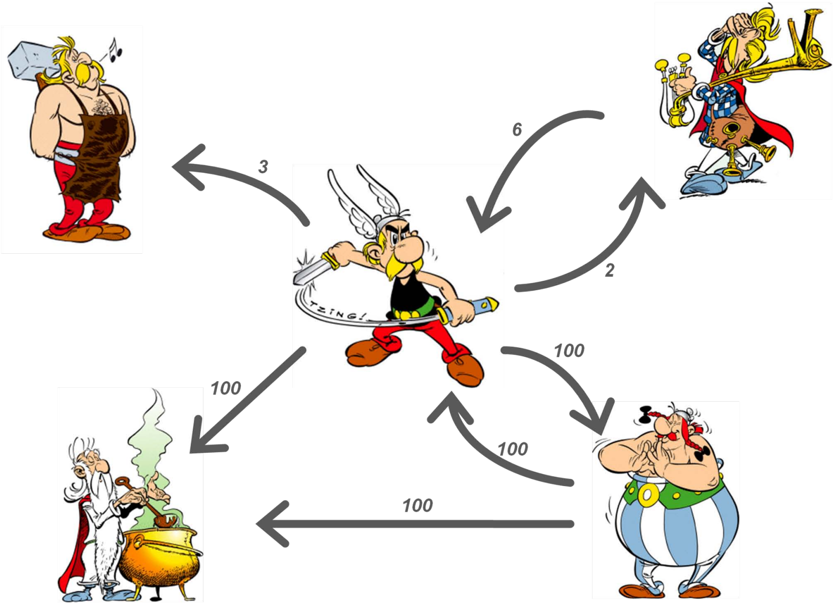

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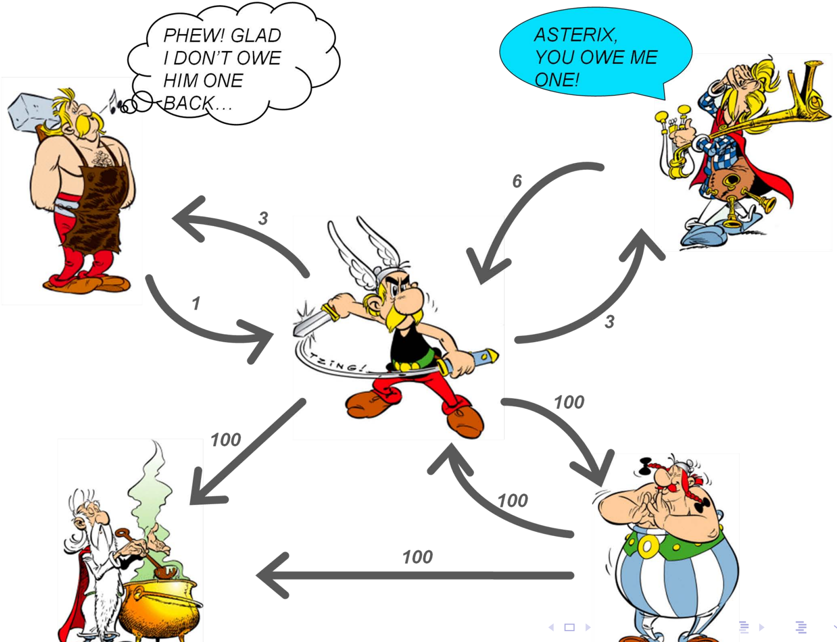

What is a Credit Network?

- Decentralized payment infrastructure introduced by [DeFigueiredo, Barr, 2005] and [Ghosh et. al., 2007]
- Do not need banks, common currency
- Models trust in networked interactions

What is a Credit Network?

- Graph $G(V, E)$ represents a network (social network, p2p network, etc.)
- **Nodes:** (non-rational) agents/players; print their own currency
- **Edges:** credit limits $c_{uv} > 0$ extended by nodes to each other[1]
- Payments made by passing IOUs along a chain of trust
- Credit gets replenished when payments are made in the other direction

[1] assume all currency exchange ratios to be unity

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- Barter/Exchange economies like P2P networks.
- Combating social spam (Facebook, LinkedIn)
- Distributing proxy addresses to circumvent censorship in repressive regimes

- Edges have integer capacity $c > 0$
- Transaction rate matrix $\Lambda = \{\lambda_{uv} : u, v \in V, \lambda_{uu} = 0\}$
- Repeated transactions; at each time step choose (s, t) with prob. λ_{st}
- Try to route a unit payment from s to t via the shortest feasible path; **update edge capacities** along the path
- Transaction fails if no path exists

Markov Chain

- Repeated transactions induce a Markov chain $\mathcal{M}$ with $(c+1)^m$ states
- State $\mathcal{S}$ of $\mathcal{M}$ captures the states of all edges in G
- Transition probability $P(\mathcal{S}, \mathcal{S}') = \lambda_{st}$, where $s \to t$ in $\mathcal{S}$ leads to $\mathcal{S}'$
- $P(\mathcal{S}, \mathcal{S})$:= failure prob. at state $\mathcal{S}$

Questions

- Steady-state distribution?
- Steady-state transaction success probability?
- Comparison with a centralized payment infrastructure

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- Success probability independent of path along which transactions are routed
- For symmetric transaction rates, the success probability for
  - **Complete Graphs:** Goes to one with increase in network size or credit capacity.
  - $G_c(n, p)$ **networks** ($p > \ln n/n$)**:** Goes to one with increase in one of n, p or c keeping the other two constant.
  - **PA networks:** Goes to one with increase in avg. node degree or credit capacity (indepedent of network size).
- Success probability in Complete graphs and Erdös-Rényi graphs only constant-factor worse than equiv. centralized payment system.

Main Results

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Definition

Let $\mathcal{S}$ and $\mathcal{S}'$ be two states of the network. We say that $\mathcal{S}'$ is **cycle-reachable** *from $\mathcal{S}$ if the network can be transformed from state $\mathcal{S}$ to state $\mathcal{S}'$ by routing a sequence of payments along feasible cycles (i.e. from a node to itself along a feasible path).*

Transactions along a feasible cycle are "free".

Theorem

Let $(s_1, t_1), (s_2, t_2), \ldots, (s_T, t_T)$ be the set of transactions of value $v_1, v_2, \ldots, v_T$ respectively that succeed when the payment is routed along the shortest feasible path from s_i to t_i. Then the same set of transactions succeed when the payment is routed along any feasible path from s_i to t_i.

Proof Sketch.

Proof by induction on T.

$\mathcal{S}_k :=$ state of the network when transactions $(s_1, t_1), \ldots, (s_k, t_k)$ are routed along the shortest feasible path

$\mathcal{S}'_k :=$ state of the network when not all of the transactions $(s_1, t_1), \ldots, (s_k, t_k)$ are routed along the shortest feasible path

From $\mathcal{S}'_k$ undo transactions $(s_k, t_k), (s_{k-1}, t_{k-1}), \ldots, (s_1, t_1)$ and redo $(s_1, t_1), \ldots, (s_k t_k)$ along their shortest feasible paths. This results in state $\mathcal{S}$.

But undoing and redoing is equal to k transactions along cycles. Therefore, $\mathcal{S}_k$ and $\mathcal{S}'_k$ are cycle-reachable.

So if (s_{k+1}, t_{k+1}) is feasible in state $\mathcal{S}_k$, it is also feasible in state $\mathcal{S}'_k$. □

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Cycle-reachability induces a partition $\mathcal{C}$ on the set of states in $\mathcal{M}$.

FACT

For any equivalence class $C \in \mathcal{C}$, if a transaction (s, t) is feasible in some state $\mathcal{S} \in C$, it is feasible for all states $\mathcal{S}' \in C$ (since $\mathcal{S}$ is cycle-reachable from $\mathcal{S}'$).

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If a transaction (s, t) is feasible in two states $\mathcal{S}_i, \mathcal{S}_j \in C$ and results in transitions to states $\mathcal{S}_i'$ and $\mathcal{S}_j'$ respectively, then $\mathcal{S}_i'$ and $\mathcal{S}_j'$ are cycle-reachable (in other words, belong to the same equivalence class).

COROLLARY

If a transaction (s, t) in some state in the equivalence class C_i results in a transition to a state in equivalence class C_j, then the reverse transaction (t, s) from any state in C_j will result in a transition to a state in C_i.

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Theorem

Consider a Markov chain $\mathcal{M}_{\mathcal{S}_0}$ starting in state $\mathcal{S}_0$ induced by a symmetric transaction rate matrix Λ. Let $\mathcal{C}_{\mathcal{S}_0} \subseteq \mathcal{C}$ be the set of equivalence classes accessible from $\mathcal{S}_0$ under the regime defined by Λ. Then $\mathcal{M}_{\mathcal{S}_0}$ has a uniform steady-state distribution over $\mathcal{C}_{\mathcal{S}_0}$.

Proof.

$\mathcal{T}_{ij} := \{(s,t) \mid s \to t \text{ in state } \mathcal{S} \in C_i \text{ leads to state } \mathcal{S}' \in C_j\}$

Define transition probability between $C_i, C_j \in \mathcal{C}_{\mathcal{S}_0}$ as

$$P(C_i, C_j) = \sum_{(s,t) \in \mathcal{T}_{ij}} \lambda_{st}$$

Since $(s,t) \in \mathcal{T}_{ij} \Leftrightarrow (t,s) \in \mathcal{T}_{ji}$ and Λ is symmetric, therefore P is a symmetric stochastic matrix.

$\Longrightarrow$ uniform distribution over $\mathcal{C}_{\mathcal{S}_0}$ is stationary w.r.t. P. □

Corollary

If $\mathcal{M}$ is an ergodic Markov chain induced by a symmetric transaction rate matrix Λ, it has a uniform steady state distribution over $\mathcal{C}$.

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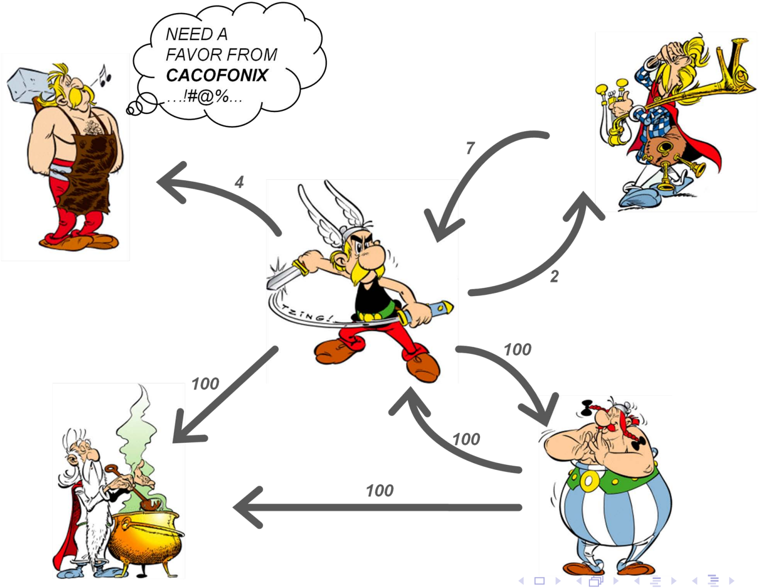
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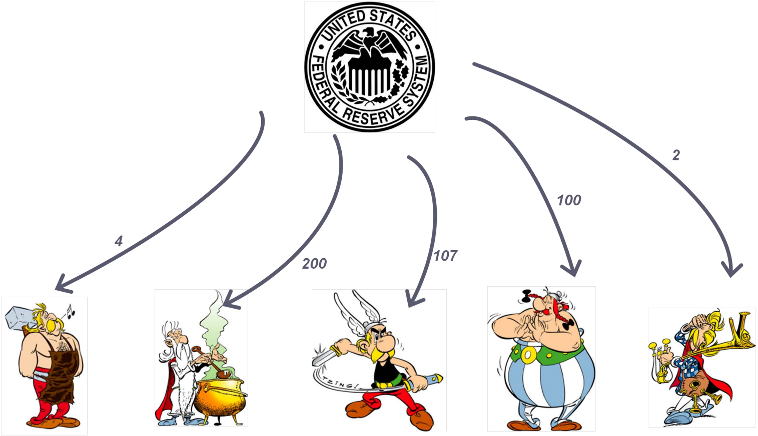
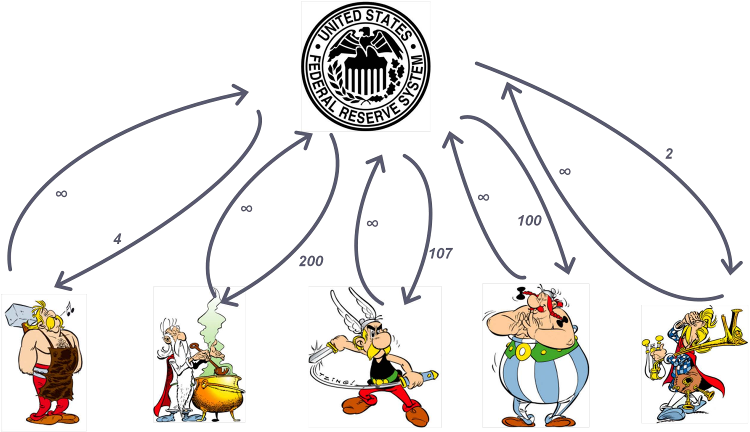


UNITED STATES
FEDERAL RESERVE SYSTEM
4
200
107
100
2



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TZING!

Convert Credit Network → Centralized Model

$$\forall u, c_{ru} = \sum_{v} c_{vu}$$

$\implies$ Total credit in the system is conserved during conversion

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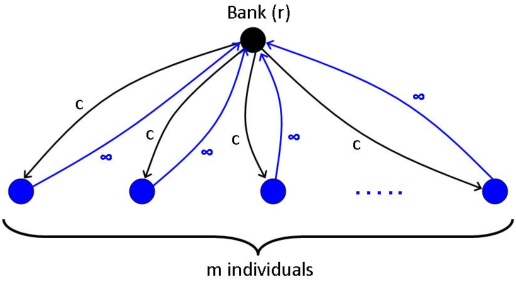
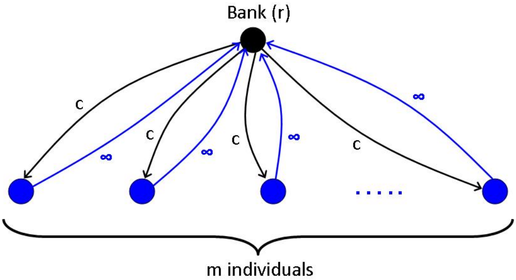


$\mathcal{M}$ has $\binom{mc+m-1}{m-1}$ states



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	Credit Network	Centralized System
Star-network	$\Theta(1/c)$	$\Theta(1/c)$
Complete Graph	$\Theta(1/nc)$	$\Theta(1/nc)$
$G_c(n, p)$[2]	$\Theta(1/npc)$	$\Theta(1/npc)$

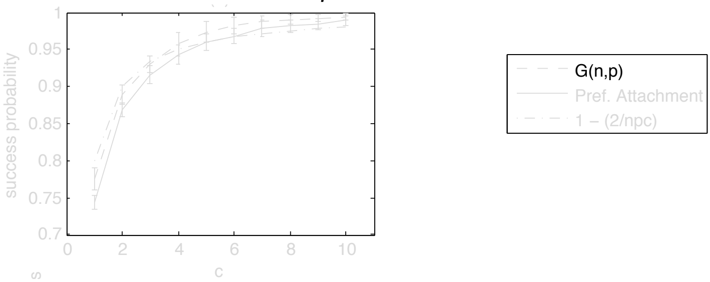
Table: Steady-state Failure Probability in Credit Network v/s Centralized System

[2] bankruptcy probability

SETUP

- Repeated transactions on $G_c(n, p)$ and PA graphs.
- Stopping criterion: success-rate in consecutive time windows $\leq \epsilon$
- Studied effect of varying network size, network density, and credit capacity
- For each run, recorded following metrics:
  - Number of (weakly) connected components
  - Avg. path length of successful transactions
  - Number of "sink" / "source" nodes
- Averaged metrics over 100 runs

$n = 100; p = 0.10; d = 5$


1
0.95
0.9
0.85
0.8
0.75
0.7
success probability
s
0
2
4
6
8
10
c
G(n,p)
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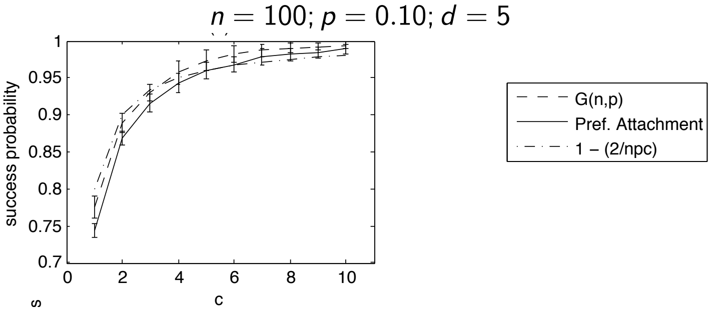
1 – (2/npc)



$n = 100; p = 0.10; d = 5$
success probability
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0.95
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0.85
0.8
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8
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c
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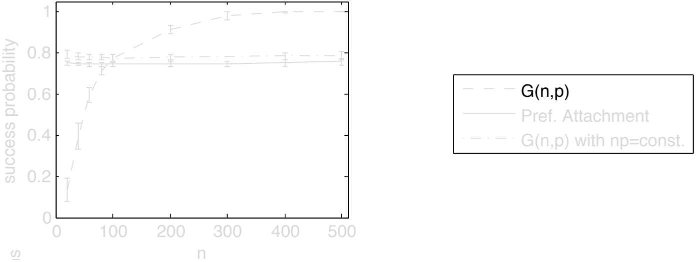
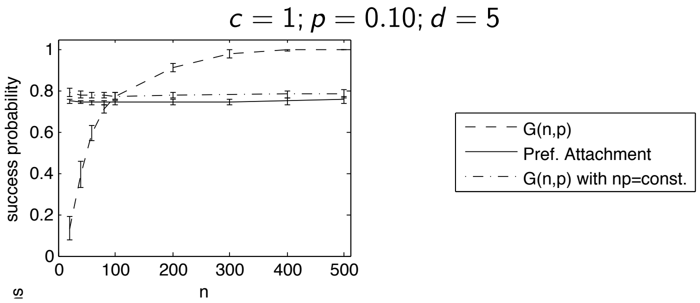
Pref. Attachment
1 – (2/npc)

$c = 1; p = 0.10; d = 5$


success probability
n
1
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
G(n,p)
Pref. Attachment
G(n,p) with np=const.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- Effect of node failures on liquidity and how it varies with network topology
- Effect of non-zero payment routing fees on liquidity
- Endow nodes with rationality: how do nodes initialize and update trust values?

Questions?

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Questions?

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