

SOBA: Secrecy-preserving Observable Ballot-level Audit

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Abstract

SOBA is an approach to election verification that provides observers with justifiably high confidence that the reported results of an election are consistent with an audit trail (“ballots”), which can be paper or electronic. SOBA combines three ideas: (1) publishing cast vote records (CVRs) separately for each contest, so that anyone can verify that each reported contest outcome is correct, if the CVRs reflect voters’ intentions with sufficient accuracy; (2) shrouding a mapping between ballots and the CVRs for those ballots to prevent the loss of privacy that could occur otherwise; (3) assessing the accuracy with which the CVRs reflect voters’ intentions for a collection of contests while simultaneously assessing the integrity of the shrouded mapping between ballots and CVRs by comparing randomly selected ballots to the CVRs that purport to represent them. Step (1) is related to work by the Humboldt County Election Transparency Project, but publishing CVRs separately for individual contests rather than images of entire ballots preserves privacy. Step (2) requires a cryptographic commitment from elections officials. Observers participate in step (3), which relies on the “super-simple simultaneous single-ballot risk-limiting audit.” Step (3) is designed to reveal relatively few ballots if the shrouded mapping is proper and the CVRs accurately reflect voter intent. But if the reported outcomes of the contests differ from the outcomes that a full hand count would show, step (3) is guaranteed to have a large chance of requiring all the ballots to be counted by hand, thereby limiting the risk that an incorrect outcome will become official and final.

1 Introduction and background

The majority of Americans now vote electronically, either on machine-counted paper ballots or on Direct Recording Electronic (DRE) machines. Electronic voting offers advantages over hand counts and lever machines, but it poses challenges for determining whether

votes were recorded and counted correctly. A wide range of security vulnerabilities and other flaws have been documented in contemporary voting equipment. The 2007 “Top-to-Bottom Review” of the systems used in California found that all the systems had “serious design flaws” and “specific vulnerabilities, which attackers could exploit to affect election outcomes” [Bowen, 2007]. While some of these vulnerabilities can be mitigated, the underlying verification challenge is formidable. As Rivest and Wack comment, “complexity is the enemy of security,” and demonstrating that any complex system is free of faults may be impossible or infeasible [Rivest and Wack, 2006].

Electronic voting systems have failed in real elections. In the 2004 general election in Carteret County, North Carolina, over 4,000 votes were lost irretrievably due to a programming error that affected UniLect Patriot voting machines, casting doubt on a statewide election outcome [Bonner, 2004]. More controversially, in the 2006 general election, ES&S iVotronic DREs in Sarasota County, Florida did not record a vote for U.S. House for about 15% of voters—far more than can plausibly be attributed to intentional undervoting. Inadvertent undervotes were probably decisive in that contest [Ash and Lamperti, 2008; Mebane and Dill, 2007]. Hypotheses explaining these undervotes include voter confusion caused by poor ballot layout [Frisina et al., 2008] and machine failure [Garber, 2008; Mebane, 2009]. Unfortunately, the forensic evidence generated by the voting systems was inadequate to determine the cause of the undervotes or the intentions of the voters.

Voter-marked paper ballots provide a clearer record of what voters did and more evidence about voter intent, but by themselves do not solve the election verification problem. In 2005, Harri Hursti repeatedly demonstrated the ability to “hack” optical scan counts when given access to a memory card [Zetter, 2005]. In a June 2006 primary election in Pottawattamie County, Iowa, incorrectly configured optical scanners miscounted ab-

sente ballot in every contest, altering two outcomes. The county auditor ordered a hand recount, which corrected the errors [Flaherty, 2006]. Similar errors in other elections may have altered outcomes without ever being detected. Even when scanners work correctly, their results may differ materially from voter intent. Consider the 2006 U.S. Senate contest in Minnesota, where Al Franken beat Norm Coleman in a hand recount largely because of ballots where the human interpretation differed from the machine interpretation.¹

1.1 Software independence

Computerized election equipment cannot be infallible, so Rivest and Wack [2006] and Rivest [2008] suggest that voting systems should be software-independent. A voting system is *software-independent* “if an undetected change or error in its software cannot cause an undetectable change or error in an [apparent] election outcome.” This idea can be generalized to define independence from hardware and from elections personnel, leading to so-called *end-to-end verifiable* election technologies. However, end-to-end technology may require fundamental changes in current voting processes.

The outcome of a contest is the set of winners, not the exact vote counts. The *apparent* outcome of a contest is the winner or winners according to the voting system. The *correct* outcome of a contest is the winner or winners that a full hand count of the “audit trail” would find. The audit trail is assumed to be an indelible record of how voters cast their votes. It might consist of a combination of voter-marked paper ballots, voter receipts, a voter-verifiable paper audit trail (VVPAT), and suitable electronic records.

This definition of “correct” is generally a matter of law. It does not necessarily imply that the audit trail is inviolate (nor that the outcome according to the audit trail is the same as the outcome according to how voters originally cast their ballots); that there is no controversy about which records in the audit trail reflect valid votes; that human observers agree on the interpretation of the audit trail; that the actual hand counting is accurate; nor that repeating the hand count would give the same answer. If there is no audit trail, defining what it means for the apparent outcome to be correct requires hypothetical counterfactuals—but for the fault in the voting system, what would the outcome have been?

Software independence means that errors that cause apparent outcomes to be wrong leave traces in the audit trail. But software independence does not guarantee any of the following:

1. that no such traces will occur if the apparent outcome is correct²
2. that those traces will be noticed or acted upon
3. that the cost of looking through the audit trail for those traces is affordable
4. that, in principle, there is a way to correct the apparent outcome without holding another election
5. that, in practice, the audit trail was preserved and protected well enough to determine the outcome according to how the voters originally cast their ballots

The penultimate property is guaranteed by strong software independence. Rivest and Wack [2006] and Rivest [2008] define a voting system to be *strongly software-independent* if an undetected change or error in its software cannot cause an undetectable change or error in an [apparent] election outcome, and moreover, a detected change or error in an [apparent] election outcome (due to change or error in the software) can be corrected without re-running the election. Having an audit trail does not guarantee that anyone will dig through it to see whether there is a problem or to correct the outcome if the outcome is wrong. Strong software independence does not correct anything, but it is an essential ingredient for a system to be self-correcting.

Compliance audits can be used to assess whether the last property listed above holds: Given that the election used a strongly software-independent voting system, did it adhere to procedures that should keep the audit trail sufficiently accurate to reconstruct the outcome according to how voters cast their ballots? Strong evidence that such procedures were followed is strong evidence that the legally correct outcome—what a full hand count of the audit trail would show—is the same as the outcome according to how the voters originally cast their ballots. As we discuss below in section 4, we believe that compliance audits should always be required: If the election fails the compliance audit,³ there is no assurance that even a full hand count of the audit trail would show the outcome according to how the voters really voted. Below, we assume that the election has passed a compliance audit.

²False alarms are possible. An analogy is that if a tamper-evident seal shows that a package has been opened, it does not follow that the package contents have been altered.

³“Failure” means failure to find strong evidence that such procedures were followed, rather than finding evidence that such procedures were not followed.

¹The 2000 presidential election may have been decided by differences between the machine interpretation of certain Florida optical scan ballots and the likely human interpretation [Keating, 2002].

1.2 Vote tabulation audits

Vote tabulation audits compare reported vote subtotals for subsets of ballots (“audit units”) with hand counts of the votes for each of those subsets. Audit units have to be subsets for which the voting system reports vote subtotals. Most present U.S. audits use audit units that consist of all the ballots cast in individual precincts or all the ballots tabulated on individual voting machines. Generally, audit laws do not have provisions that would lead to correcting incorrect electoral outcomes [Hall et al., 2009].⁴

A *risk-limiting post-election audit* uses the audit trail to guarantee that there is a large, pre-specified probability that the audit will correct the apparent outcome if the apparent outcome is wrong. Risk-limiting audits are widely considered best practice [Lindeman et al., 2008]. Risk-limiting audits have been endorsed by the American Statistical Association [American Statistical Association, 2010], the Brennan Center for Justice, Common Cause, the League of Women Voters, and Verified Voting, among others. California AB 2023 (2010), requires a pilot of risk-limiting audits in 2011 [Saldaña, 2010]. Colorado Revised Statutes §1-7-515 calls for implementing risk-limiting audits by 2014.

The first method for conducting risk-limiting audits was proposed by Stark [2008a]; numerous improvements have been made [Stark, 2008b, 2009b,d,c; Miratrix and Stark, 2009; Stark, 2010b]. See also [Checkoway et al., 2010]. Risk-limiting audits limit the risk of failing to correct an outcome that is wrong. The risk limit is 100% minus the minimum chance that the audit corrects the outcome. If the outcome is correct in the first place, a risk-limiting audit cannot make it wrong; but if the outcome is wrong, a risk-limiting audit has a large chance of correcting it. Hence, the probability that the outcome according to a risk-limiting audit is the correct outcome is at least 100% minus the risk limit.

For systems that are strongly software-independent, adding a risk-limiting audit addresses the second condition above: It ensures a large, pre-specified probability that the traces will be noticed and will be used to correct the apparent outcome if the apparent outcome is wrong.

1.3 Our goal

Our goal in this work is to sketch a personally verifiable privacy-preserving P -resilient canvass framework. We must first say what this means.

A *canvass framework* consists of the vote-tabulation system together with other human, hardware, software, and procedural components of the canvass, including

⁴For instance, under New York law, each county determines independently whether its audit in a particular contest must be expanded. This provision means that a correct outcome might be changed to an incorrect outcome even if the conduct of the audit is formally flawless.

compliance and vote-tabulation audits. A canvass framework is *resilient with probability P* or *P -resilient* if the probability that the outcome it gives⁵ is the correct outcome is at least P , even if its software has an error, shortcoming, or undetected change.⁶ Resilience means that the framework tends to recover from faults. If a canvass framework is P -resilient, either the outcome it gives when all is said and done is correct, or something occurred that had probability less than $1 - P$. The canvass framework that results from performing a risk-limiting audit on a strongly software-independent voting system that passes a compliance audit is P -resilient, with P equal to 100% minus the risk limit. If the system fails the compliance audit, the framework should not declare any outcome. Instead, the election should be re-run.

Even if a canvass framework is P -resilient, in practice the public might not trust the system unless they can observe crucial steps, especially the audit. The mere right or opportunity to observe the audit will not engender much trust if—as a practical matter—no single person or small group *could* observe all the steps that are essential to ensuring the accuracy of the final result. For instance, if a vote-tabulation audit takes ten teams of auditors working in separate offices four days to complete, it would take a large team of independent observers—with lots of free time and long attention spans—to verify that the audit was carried out correctly. The longer an audit takes and the more people required to carry out the audit, the more opportunities there are to damage the audit trail, and the harder it is for an observer to be satisfied that the audit has been conducted correctly.

We define a canvass framework to be *personally verifiable P -resilient* if it is P -resilient and a single individual could, as a practical matter, observe enough of the process to have convincing evidence that the canvass framework is in fact P -resilient.

The transparency required for a canvass framework to be personally verifiable can impact privacy. For instance, publishing images of all the ballots cast in an election⁷ might give the individuals compelling evidence that the vote tabulation system found the correct outcome, since the images allow people to count the votes themselves—at least to the extent that voter intent is unambiguous.⁸ But publishing ballot images can facilitate vote-selling

⁵As discussed in section 4, to be P -resilient, a canvass framework should refrain from giving any outcome at all if some preconditions are not met.

⁶The probability comes from the overall voting system, in our case from the fact that the audit relies on a random sample. The probability does not come from treating votes, voters, or election outcomes as random, for instance.

⁷There also needs to be proof that the images are sufficiently complete and accurate to determine the correct outcome.

⁸Verification methods like Humboldt County Election Transparency Project (see below) involve publishing digital images of all the ballots.

and coercion and can compromise privacy, because voters can deliberately or accidentally reveal their identities through marks on the ballots including idiosyncrasies of how individuals fill in bubbles [Calandrino et al., 2011] or even the fiber structure of the paper on which the ballot is printed [Calandrino et al., 2009].⁹

A lesser but substantial degree of transparency is conferred by publishing cast vote records (CVRs)¹⁰ enabling anyone to verify that the contest outcomes are correct—if the CVRs are accurate. However, as Popoveniuc and Stanton [2007] and Rescorla [2009] point out, publishing CVRs also can aid vote-selling or coercion because of the potential for pattern voting. One typical sample ballot (from Tulsa, Oklahoma) contains 18 contests with over 589,000 possible combinations if a voter votes in every contest, or over 688 million combinations allowing for undervotes. Thus, a voter could be instructed to vote for the preferred candidate in one contest, and to cast a series of other votes that would almost certainly (especially within a precinct), confirm the voter’s identity if all of the voter’s selections were published. Hence, publishing whole-ballot CVRs for large numbers of ballots improves transparency but can sacrifice privacy.

When there is not strong evidence that the apparent outcome is correct, risk-limiting audits can require examining the entire audit trail, potentially exposing all the ballots to public scrutiny.¹¹ If the apparent outcome is wrong, such exposure is necessary in order to correct the outcome. Therefore, if a risk-limiting audit is to be personally verifiable, there may be occasions where compromising privacy is unavoidable. But minimizing the number of ballots or whole-ballot CVRs that are routinely exposed helps protect privacy, impeding vote-selling and coercion.

We define a canvass framework to be *personally verifiable privacy-preserving P-resilient* if it is personally verifiable *P-resilient* and it does not sacrifice privacy unnecessarily. Neither *personally verifiable* nor *privacy-*

preserving is a mathematically precise characteristic, while *P-resilience* is.

The contribution of the present work is to sketch a personally verifiable privacy-preserving *P-resilient* voting system. We assume, as a foundation for building this system, that we are starting with a strongly software-independent voting system with an audit trail that corresponds to individual ballots. Moreover, we assume that a compliance audit has determined that the audit trail generated by the system is sufficiently trustworthy to reflect the correct outcomes of the contests. We augment the system with procedures and data structures that make it possible for an individual observer to gain compelling evidence that either the outcomes are correct, or something very unlikely occurred—that is, that the overall canvass framework is *P-resilient*. Unless some of the apparent outcomes are wrong or a margin is extremely small, gathering that evidence will generally involve exposing only a tiny percentage of ballots and whole-ballot CVRs.

In essence, our method adds a special risk-limiting audit to a strongly software-independent voting system (one that has had a compliance audit to ensure that its audit trail is intact). Since one person cannot be in two places at the same time, the procedure cannot be personally verifiable if it involves auditing a multi-jurisdictional contest in different jurisdictions simultaneously; it would then be necessary to trust confederates to observe what is happening elsewhere. The next few sections outline elements of this risk-limiting audit.

2 Ballot-level risk-limiting audits

One key to keeping the process personally verifiable (by keeping amount of observation required low) and to protecting privacy (by exposing as few ballots as possible to observers) is to audit the record at the level of individual ballots, rather than large batches of ballots such as precincts. The fewer ballots there are in each audit unit, the smaller the expected counting burden for risk-limiting audits tends to be—when the electoral outcome is correct (see, e.g., [Stark, 2009a, 2010a,b]). A vote-tabulation audit based on checking the CVRs of individual ballots against a human interpretation of those ballots is often called a “ballot-level audit,” a “single-ballot audit,” or a “ballot-based audit.” Because they reduce the time it takes to audit and the number of ballots involved, ballot-level risk-limiting audits are especially amenable to personal verification.

Ballot-level audits are extremely efficient statistically, but they are not simple to implement using current voting systems. To perform a ballot-level audit, there must be a way to identify each ballot uniquely, for instance, a serial number on a paper ballot, or identifying the bal-

⁹There are arguments that images of ballots should be published anyway—that transparency is more important than privacy. In jurisdictions that permit voting by mail, there is an opportunity to confirm how someone votes for the purpose of vote-selling or coercion; indeed, someone could fill out another’s ballot. Whether publishing images of ballots would change the rate of vote-selling or coercion substantially is the subject of some debate.

¹⁰In the 2002 FEC Voting System Standards [Federal Election Commission, 2002], these were called “ballot images”; however, the term CVR has been used in more recent EAC Voluntary Voting System Guidelines [Election Assistance Commission, 2005]; we prefer the latter term because it does not suggest an actual image but rather a record of the interpretation of the system’s interpretation of the ballot. And what matters is the system’s interpretation of the ballot as a set of votes.

¹¹One could have a risk-limiting audit that, if it had not terminated after some fraction of the ballots had been examined, triggered a hand count of the remaining ballots, but did not allow the public to observe that hand count. But then why should the public trust that the hand count was accurate?

lot by its location: “the 17th ballot in deck 152 scanned by scanner C,” for instance.¹² There must also be a way to match each ballot to its CVR. Some commercial voting systems do not generate or do not store CVRs for individual ballots. Other voting systems record individual CVRs, but are designed make it difficult or impossible to match individual CVRs to the ballots they purport to represent. In some cases, audit trails have identifiers that can be used to find the corresponding CVRs; this method was used for part of a 2008 audit in Eagle County, Colorado [Branscomb, 2008]¹³ and a ballot-level risk-limiting audit in Orange County, California, in 2011 [P.B. Stark, personal communication, 2011]. However, to protect privacy, most paper ballots do not have identification numbers. In a 2009 pilot ballot-level audit in Yolo County, California, Stark [2009c] exploited the fact that the CVRs and the physical ballots were in the same order. The scanned images associated with each CVR in the audit sample were compared with the physical ballots to check the accuracy of the CVRs.

Calandrino et al. [2007] describe an approach to election verification that involves imprinting ballots with identification numbers and scanning the ballots with a “parallel” system in addition to the system of record. The parallel system derives its own CVRs, from which the apparent contest outcome can be determined independently. The accuracy of the unofficial CVRs and of the imprinting process is then assessed by a ballot-level audit.

Since 2008, the Humboldt County Election Transparency Project (Humboldt County ETP) has experimented with publishing ballot images and independently tabulating CVRs extracted from those images. Using commercially available equipment, Humboldt County ETP rescans paper ballots after embossing them with serial numbers. Then, open-source software is used to form CVRs from the digital images. Humboldt County ETP has processed ballots for six elections and published scanned ballot images as well as its version of the CVRs for some of them. The results based on their re-scans generally have agreed well with the original results, with one important exception: The Humboldt County ETP analysis of the November 2008 election uncovered a defect in the election management software that led the results of an entire ballot batch to be silently discarded!

¹²If an identifier is printed on paper ballots, the printing should occur after the voter casts his or her vote and the ballots are co-mingled. If the identifier is printed before the voter casts his or her vote, privacy could be compromised.

¹³Optical-scan ballots as well as DRE paper audit trails can have identifiers. For instance, in Boulder County, Colorado, the Hart Ballot Now system is configured to print unique identifiers and bar codes on each ballot. In Orange County, California, ballots for the Hart Ballot Now system have non-unique identifiers and bar codes (numbered 1–2500, then repeating).

The Clear Ballot Group, inspired in part by Humboldt County ETP, is developing a system that, in its words, could permit election outcomes to be “thoroughly and transparently verified within 36–48 hours after the polls close.” Neither the Humboldt County ETP nor Clear Ballot Group currently incorporate risk-limiting audits,¹⁴ but the parallel scans their systems perform facilitate ballot-level risk-limiting audits, along the general lines proposed by Calandrino et al. [2007]. If the system of record and the parallel system agree on the set of winners, a risk-limiting audit of the parallel system transitively confirms the outcome according to the system of record.¹⁵

3 A privacy-preserving audit

The method we propose here presupposes that CVRs are available, either from the system of record or from a parallel system. It publishes all the data contained in the CVRs in a form that (1) still permits all observers to check the contest outcomes on the assumption that the CVRs are accurate, (2) does not compromise privacy, and (3) enables the CVRs to be checked against the audit trail while minimizing the loss of privacy.

In SOBA, election officials make a cryptographic commitment¹⁶ to the full set of CVRs by publishing the CVRs separately for each contest, disaggregating the ballots (we call these contest-CVRs or CCVRs in contrast to whole-ballot CVRs), and a shrouded link between each CCVR and the ballot it purports to represent. Splitting the CVRs into CCVRs and obfuscating the identity of the ballot from which each CCVR comes eliminates some of the information required to identify a voter’s ballot style or to use pattern voting to signal the voter’s identity.¹⁷ This makes the procedure privacy-preserving. But it retains enough information for any observer to check that the apparent outcome agrees with the outcome according to the CCVRs, for each contest. That is, there is a known algorithm (the winner algorithm¹⁸) that observers

¹⁴Clear Ballot Group is adding support for risk-limiting audits to their software [L. Moore, personal communication, 2011].

¹⁵This is true as long as the systems agree on the set of winners, even if they disagree about vote totals or margins. For instance, suppose candidate A defeats candidate B by one percentage point in the original returns, and by ten points according to the parallel system. Such a large discrepancy might justify close scrutiny, but a risk-limiting audit of the results of the parallel system would still provide strong evidence that A defeated B, or would lead to a full hand count to set the record straight.

¹⁶See http://en.wikipedia.org/wiki/Commitment_scheme. Cryptographic commitments have two important properties, the binding property and the hiding property, discussed in section 3.2.

¹⁷Of course, if there is a contest in which few voters are eligible to vote, eligibility itself is a signal.

¹⁸For first-past-the-post contests, the winner algorithm just finds who has the most votes. Other voting schemes, such as instant-runoff voting (IRV) or ranked choice voting (RCV), have more complicated

can apply to the published CCVRs to calculate the correct outcome of every contest—provided the CCVRs reflect the ballots (more generally, audit trail) accurately enough. This is part of making the procedure personally verifiable. Loosely speaking, the required level of accuracy depends on the number of CVRs that must have errors for the apparent outcome to be wrong:¹⁹ The fewer ballots that need to be changed to affect the outcome, the larger the sample generally will need to be to attain a given level of confidence that the apparent outcome is correct.

The CCVRs might fail to be sufficiently accurate because

- At least one CCVR and the ballot it purports to represent do not match because human and machine interpretations of voter intent differ (for instance, because the voter marked the ballot improperly). This is a failure of the generation of CCVRs.
- At least one CCVR does not in fact correspond to any ballot. It is an “orphan.” This is a failure of the mapping between ballots and CCVRs.
- More than one CCVR for the same contest is mapped to the same ballot. It is a “multiple.” This is also a failure of the mapping between ballots and CCVRs.
- There is no CCVR corresponding to some voting opportunity on a ballot.

A failure of the mapping might be the more distressing source of error, since it is a failure on the part of the election official, but we must ensure (statistically) that—together—all sources of error did not combine to cause the outcome to be wrong. SOBA uses a risk-limiting audit to assess statistically whether the winners according to the full audit trail differs from the winners according to the CCVRs, for all contests under audit, taking into account all sources of error. If the outcome according to the CCVRs is incorrect, the audit is very likely to proceed to a full hand count of the audit trail, thereby revealing the correct outcome. This provides P -resilience.

To make the risk-limiting audit possible, elections officials are required to publish another file, the *ballot style file*, which contains ballot identifiers and lists the contests each of those ballots contains. It does not contain the voters’ selections.

The risk-limiting technique we propose is the *super-simple simultaneous single-ballot risk-limiting au-*

winner algorithms.

¹⁹In plurality voting, this is the margin or the set of margins between each (winner, loser) pair. Defining the margins for IRV and calculating them for a given set of reported results is not simple. See Cary [2011]; Magrino et al. [2011].

dit [Stark, 2010b]. It is not the most efficient ballot-level audit, but the calculations it requires can be done by hand, increasing transparency. It involves drawing ballots at random with equal probability; some more efficient audits require using different probabilities for different ballots, which is harder to implement and to explain to the public. Moreover, this technique allows a collection of contests to be audited simultaneously using the same sample of ballots. That can reduce the number of randomly selected ballots that must be located, interpreted, and compared with CVRs, decreasing the cost and time required for the audit and thereby increasing transparency.

The following subsections give more technical detail.

3.1 Data framework and assumptions

We assume that the audit trail consists of one record per ballot cast. There are C contests we wish to assess. The contests might be simple measures, measures requiring a super-majority, multi-candidate contests, or contests of the form “vote for up to W candidates.”²⁰ We refer to records in the audit trail as “ballots.” A ballot may be an actual voter-marked paper ballot, a voter-verifiable paper audit trail (VVPAT), or a suitable electronic record.

There are N ballots in the audit trail that each contain one or more of the C contests. Each ballot can be thought of as a list of pairs, one pair for each contest on that ballot. Each pair identifies a contest and the voter’s selection(s) in that contest, which might be an undervote or a vote for one or more candidates or positions. Examining a ballot by hand reveals all the voter’s selections on that ballot; we assume that there is no ambiguity in interpreting each voter’s intentions from the audit trail.

Before the audit starts, the voting system must report results for each of the C contests. The report for contest c gives N_c , the total number of ballots cast in contest c (including undervotes and spoiled ballots), as well as the number of valid votes for each position or candidate in contest c . Let $M \equiv N_1 + N_2 + \dots + N_C$ denote the total number of voting opportunities on the N ballots. We assume that the compliance audit assures us (e.g., through ballot accounting) that the reported values of N_c are accurate, and that the audit trail is trustworthy. In the present work, we do not consider attacks on the audit trail.

There is a published “ballot style file.” Each line in the ballot style file lists a ballot identifier and a list of contests that ballot is supposed to contain. The ballot identifier uniquely identifies a ballot in the audit trail. The identifier could be a number that is printed on a paper

²⁰We do not specifically consider instant-runoff voting or ranked-choice voting here. Risk-limiting methods can be extended to such voting methods, but the details are complex.

ballot or unambiguous instructions for locating the ballot (e.g., the 275th ballot in the 39th deck). There should be N lines in the file, and the N ballot identifiers should be unique. Because the ballot style file is published, individual can check this for themselves. Moreover, individuals can check whether the number of lines in the ballot style file that list contest c equals N_c , the total number of ballots the system reports were cast in contest c .

Before the audit starts, the voting system or a parallel system has produced a CVR for each ballot. These are not published as whole-ballot CVRs. Rather, the CVRs are split by contest to make contest-specific CVRs (CCVRs) that contain voters' selections in only one contest. Each whole-ballot CVR is (supposed to be) split into as many CCVRs as there are contests on the ballot.

The CCVRs for the contests are published in C files, one for each contest. The CCVR file for contest c should contain N_c lines; because this file is published, individuals can check this for themselves. Each line in the CCVR file for contest c lists a voter's selection and a shrouded version of the identifier of the ballot that the selection is supposed to represent. The order of the lines in each of the C CCVR files should be shuffled (preferably using random permutations) so that whole CVRs cannot be reassembled without knowing secret information.²¹

The public can confirm whether the contest outcomes according to the CCVR files match the voting system's reported outcomes. If they do not match, there should be a full hand count of any contests with discrepant outcomes. We assume henceforth that the outcomes do match, but we do not assume the exact vote totals according to the CCVR files match the reported vote totals.

The data include one more file that is not published, the *lookup file*. The lookup file contains M lines, one for each voting opportunity on each ballot. Each line has three entries: a shrouded ballot identifier, the corresponding unshrouded ballot identifier, and a number ("salt") that is used in computing the shrouded identifier from the unshrouded identifier using a cryptographic commitment function, as described below. (For a review of uses for cryptography in voting, see Adida [2006].)

The salt on the j th line of the file is denoted u_j . Each line corresponds to a (ballot, contest) pair: We can think of u_j as being u_{ic} , the salt used to shroud the identity of ballot b_i in the CCVR file for contest c . The election official will use this file to convince observers that every selection on every ballot corresponds to exactly one entry in a CCVR file, and vice versa.

²¹For example, each CCVR file could be sorted in order of the shrouded ballot identifier.

3.2 Shrouding

The method of shrouding ballot identifiers is crucial to the approach. SOBA requires election officials to cryptographically commit to the value of the ballot identifier that goes with each CCVR. A cryptographic commitment ensures that the ballot identifier is secret but indelible: The election official can, in effect, prove to observers that a shrouded identifier corresponds to a unique unshrouded identifier, but nobody can figure out which unshrouded identifier corresponds to a given shrouded identifier without secret information.

The next few paragraphs describe a suggested instantiation of the cryptographic commitment. We assume that ballot identifiers all have the same length. If necessary, this can be achieved by padding identifiers with leading zeros. The commitment function $H()$ must be disclosed publicly and fixed for the duration of the election.

Each commitment represents a claim about a voter's selection(s) on a given ballot in a given contest. For each set of selections that any voter made in each contest, including undervotes and votes for more than one candidate, the election official will create a set of commitments. Each commitment designates the ballot identifier of a ballot that the election official claims contains that set of selections in that contest. To commit to the ballot identifier b , the election official selects a secret "salt" value u ²² and computes the commitment value $y = H(b, u)$. At a later stage, the official can open the commitment by revealing u and b : Then anyone can verify that the value y revealed earlier is indeed equal to $H(b, u)$.

Loosely speaking, a commitment function must have two properties, the *binding property* and the *hiding property*. The binding property makes it infeasible for the official to find any pair $(b', u') \neq (b, u)$ for which $H(b', u') = H(b, u)$. This provides integrity by helping to ensure that election officials cannot contrive to have more than one CCVR for a given contest claim to come from the same ballot.²³ The binding property is crucial for P -resilience; indeed, the proof of P -resilience requires only that the commitment have the binding property and that $\{N_c\}_{c=1}^C$ are known.

The hiding property makes it infeasible for anyone with access only to the shrouded values $H(b, u)$ to learn anything about which ballot is involved in each commitment. This provides privacy by helping to ensure that observers cannot reassemble whole-ballot CVRs from the CCVR files without extra information. If observers could reassemble whole-ballot CVRs, that would open a

²²To protect voter privacy, it must be infeasible to guess the salts: Each salt should contain many random or pseudo-random bits. For the commitment to be effective, the length of all salt values should be fixed and equal. See section 4.

²³See step 7 of the proof in section 3.4.

channel of communication (pattern voting) for coercion or vote selling. Ballot identifier b may appear in multiple commitments since a separate commitment is generated for each candidate selection on each ballot. The hiding property ensures that those collections of commitments do not together reveal the value of any b . This is crucial for the method to be privacy-preserving.

An HMAC (as described in Federal Information Processing Standard Publication 198) with a secure hash function such as SHA-256 (described in Federal Information Processing Standard Publication 180-2) can be used to instantiate the commitment function. However, since each of the parameters of the commitment function is of fixed length it is more efficient to simply use a cryptographic hash function such as SHA-256 directly. The length of the ballot identifiers does not matter, as long as all ballot identifiers in the election have the same length. We recommend that all salt values have equal length, of at least 128 bits. Our results do not depend on the particular commitment function chosen, as long as it has both the binding and hiding properties.²⁴

We now describe how to perform a risk-limiting audit that simultaneously checks the accuracy of the CCVRs, whether each CCVR entry comes from exactly one ballot, and whether every voting opportunity on every ballot is reflected in the correct CCVR file.

3.3 The audit

The first three steps check the consistency of the CCVRs with the reported results and the uniqueness of the shrouded identifiers.

1. Verify that, for each contest c , there are N_c entries in the CCVR file for contest c .
2. Verify that, for each contest c , the CCVR file shows the same outcome as the reported outcome.
3. Verify that the $M = N_1 + \dots + N_C$ shrouded ballot identifiers in all C CCVR files are unique.

If step 2 shows a different outcome for one or more contests, those contests (at least) should be completely hand counted.

Steps 4 and 5 check the logical consistency of the ballot style file with the reported results.

4. Verify that, for each contest c , there are N_c entries in the ballot style file that list the contest.
5. Verify that the ballot identifiers in the ballot style file are unique.

²⁴Menezes et al. [1996] offers a thorough treatment of hash functions and their use for commitments in applications such as digital signatures.

If steps 1, 3, 4, or 5 fail, there has been an error or misrepresentation. The election official needs to correct all such problems before the audit can start.

The remaining steps comprise the statistical portion of the risk-limiting audit, which checks whether the CCVRs and the mapping from ballots to CCVRs is accurate enough to determine the correct winner.

6. Set the audit parameters:

- (a) Choose the risk limit α .
- (b) Choose the maximum number of samples D to draw; if there is not strong evidence that the outcomes are correct after D draws, the entire audit trail will be counted by hand.
- (c) Choose the “error bound inflator” $\gamma > 1$ and the error tolerance $\lambda \in (0, 1)$ for the super-simple simultaneous method [Stark, 2010b] ($\gamma = 1.01$ and $\lambda = 0.2$ are reasonable values).
- (d) Calculate

$$\rho = \frac{-\log \alpha}{\frac{1}{2\gamma} + \lambda \log(1 - \frac{1}{2\gamma})}. \quad (1)$$

- (e) For each of the C contests, calculate the margin of victory m_c in votes from the CCVRs for contest c .²⁵
- (f) Calculate the *diluted margin* μ : the smallest value of m_c/N among the C contests.²⁶
- (g) Calculate the initial sample size $n_0 = \lceil \rho/\mu \rceil$.
- (h) Select a seed s for a pseudo-random number generator (PRNG).²⁷ Observers and election officials could contribute input values to s or s could be generated by an observable, mechanical source of randomness such as rolls of a 10-sided die. The seed should be selected only once.

7. Draw the initial sample by finding n_0 pseudo-random numbers between 1 and N and audit the corresponding ballots:

- (a) Use the PRNG and the seed s to generate n_0 pseudo-random numbers, r_1, r_2, \dots, r_{n_0} .

²⁵This would be replaced by a different calculation for IRV or RCV contests. See, e.g., Magrino et al. [2011]; Cary [2011].

²⁶The diluted margin controls the sample size. If contest c has the smallest value of m_c/N and N_c is rather smaller than N , it can be more efficient to audit contest c separately rather than auditing all C contests simultaneously.

²⁷The code for the PRNG algorithm should be published so that it can be checked and so that, given the seed s , observers can reproduce the sequence of pseudo-random numbers. The PRNG should produce numbers that are statistically indistinguishable from independent random numbers uniformly distributed between 0 and 1 (i.e., have large p -values) for sample sizes up to millions for a reasonable battery of tests of randomness, such as the Diehard tests.

- (b) Let $\ell_j \equiv \lceil Nr_j \rceil$, $j = 1, \dots, n_0$. This list might contain repeated values. If so, the tests below only need to be performed once for each value, but the results count as many times as the value occurs in the list.²⁸
- (c) Find rows $\ell_1, \dots, \ell_{n_0}$ in the ballot style file.
- (d) Retrieve the ballots b_{ℓ_j} in the audit trail identified by those rows in the ballot style file. If there is no ballot with identifier b_{ℓ_j} , pretend in step 7(g) below that the ballot showed a vote for the runner-up in every contest listed in that row of the ballot style file.
- (e) Determine whether each ballot shows the same contests as its corresponding entry in the ballot style file. If there are any contests on the ballot that are not in the ballot style file entry, pretend in step 7(g) below that the CCVR for that (ballot, contest) pair showed a vote for the apparent winner of the contest. If there are any contests in the ballot style file entry that are not on the ballot, pretend in step 7(g) below that the ballot showed a vote for the apparent runner-up for that contest.
- (f) For each ballot b_{ℓ_j} in the sample, the election official reveals the value of $u_{\ell_j c}$ for each contest c on the ballot.
- (g) For each ballot in the sample, for each contest on that ballot, observers calculate $H(b_{\ell_j}, u_{\ell_j c})$ and find the entry in the CCVR file for contest c that has that shrouded identifier. If the shrouded identifier is not in the CCVR file, pretend that the CCVR file showed that the voter had selected the apparent winner of contest c . Compare the voter's selection(s) according to the CCVR file to the voter's selection(s) according to a human reading of ballot b_{ℓ_j} . Find e_{ℓ_j} , the largest number of votes by which any CCVR for ballot b_{ℓ_j} overstated the margin between any (winner, loser) pair in any contest on ballot b_{ℓ_j} . This number will be between -2 and $+2$.

8. If no ballot in the sample has $e_{\ell_j} = 2$ and no more than $\lambda \mu n_0$ have $e_{\ell_j} = 1$, the audit stops. (In this calculation, the value of e_{ℓ_j} should be counted as many times as ℓ_j occurs in the sample.)

9. Otherwise, calculate the Kaplan-Markov P -value, P_{KM} according to equation (9) in Stark [2009d,c,

²⁸The auditing method relies on sampling with replacement to limit the risk.

2010b].²⁹ If P_{KM} is less than α , the audit stops. If P_{KM} is greater than α , the sample is expanded: Another random number r_j is generated and steps 7(c)–(g) are repeated. The value of P_{KM} is updated to include the overstatement errors found in the new draw.³⁰ This continues until either $P_{KM} \leq \alpha$ or there have been D draws. In the latter case, all remaining ballots are counted by hand, revealing the true outcome.

The next section establishes that this procedure in fact gives a risk-limiting audit.

3.4 Proof of the risk-limiting property

If the ballot style file is correct and entries in the CCVR files are mapped properly to voting opportunities on actual ballots, the only potential source of error is that CCVR entries do not accurately reflect the voters' selections according to a human reading of the ballot. If that is the case, this is an "ordinary" risk-limiting audit, and the proof in Stark [2010b] that the super-simple simultaneous method is risk-limiting applies directly.

Suppose therefore that the ballot style file or the mapping between ballots and CCVRs is faulty. Recall that the super-simple simultaneous method assumes that no ballot can overstate any margin by more than 2γ votes, where $\gamma > 1$. There are seven cases to consider.

1. The ballot style file has more than one entry that corresponds to the same actual ballot, or more than one actual ballot corresponds to the same entry in the ballot style file. These faults are precluded by the uniqueness of the ballot identifiers and of the recipes for locating the actual ballot with each identifier.
2. More than one ballot identifier corresponds to the same shrouded entry (for different values of u). This is precluded by the binding property of H .

²⁹We consider only plurality voting here: IRV is more complicated. For each contest c , let \mathcal{W}_c be the indices of the apparent winners of the contest and let \mathcal{L}_c be the indices of the apparent losers of the contest. If $w \in \mathcal{W}_c$ and $x \in \mathcal{L}_c$, let V_{wx} be the margin in votes between candidate w and candidate x according to the CCVR file for contest c . For each candidate k on ballot ℓ , let $v_{\ell k}$ denote the number of votes for candidate k on ballot ℓ according to the CCVR file and let $a_{\ell k}$ denote the number of votes on ballot ℓ for candidate k according to a human reading of ballot ℓ . Let

$$\varepsilon_\ell \equiv \max_c \max_{w \in \mathcal{W}_c, x \in \mathcal{L}_c} (v_{\ell w} - a_{\ell w} - v_{\ell x} + a_{\ell x}) / V_{wx}. \quad (2)$$

Then

$$P_{KM} \equiv \prod_{j=1}^n \frac{1 - 1/U}{1 - \frac{\varepsilon_{\ell_j}}{2\gamma/\sqrt{V}}}. \quad (3)$$

³⁰Overstatements are calculated as step 7 above, including, in particular, steps 7(e) and 7(g), which say how to treat failures to find ballots or contests.

3. The ballot style file contains identifiers that do not correspond to actual ballots, or claims that a ballot contains a contest that it does not actually contain. The biggest effect this could have on an apparent contest outcome is if the ballot that entry is supposed to match showed a vote for the runner-up in every missing contest, which is no greater than a two-vote change to any margin. Because the audit samples entries of the ballot style file with equal probability, this kind of error in an entry is just as likely to be revealed as any other. If such a ballot style file entry is selected for audit, steps 7(d) and 7(e) treat it this worst-case way.
4. The ballot style file claims that a ballot does not contain a contest that it does contain. The biggest effect this could have on an apparent contest outcome is if the CCVR for that contest showed a vote for the apparent winner, which cannot change the margin by more than two votes, so the error-bound assumptions are satisfied. Because the audit samples entries of the ballot style file with equal probability, this kind of error in an entry is just as likely to be revealed as any other. If such a ballot style file entry is selected for audit, step 7(e) treats it this worst-case way.
5. There are ballots whose identifiers do not appear in the ballot style file. Since there are the same number of ballots as entries in the ballot style file and the ballot identifiers in the ballot style file are unique, there must be ballot identifiers in the ballot style file that do not match any ballot. Hence, case (3) holds.
6. There are CCVRs for which the shrouded ballot identifier is not the identifier of any ballot. If the shrouded identifier matches an identifier in the ballot style file, we are in case (3). Suppose therefore that the shrouded identifier does not match any in the ballot style file. Suppose this happens for contest c . The preliminary checks show that the ballot style file has exactly N_c entries for contest c and that there are exactly N_c entries in the CCVR file for contest c . Therefore, if there is such a CCVR, one of the ballot style file entries that lists contest c has an identifier that does not occur in shrouded form in the CCVR file for that contest. The largest effect this could have on contest c is if the “substituted” CCVR entry reported a vote for the apparent winner; this cannot overstate the margin by more than two votes, so the audit’s error-bound assumption still holds. Because the audit samples entries of the ballot style file with equal probability, this kind of error in a ballot style file entry is just as likely to be revealed as any other. If such a ballot style

file entry is selected for audit, step 7(e) treats it this worst-case way.

7. The same ballot identifier appears in shrouded form more than once in a single CCVR file. As in the previous case, we know there are N_c entries in the CCVR file for contest c and N_c entries in the ballot style file that include contest c ; moreover, the identifiers in the ballot style file are unique. Hence, there must be at least one entry in the ballot style file that lists contest c for which the ballot identifier does not appear in shrouded form in the CCVR file. We are therefore in case (6).

4 Discussion

Others have proposed election verification methods that involve a cryptographic commitment by elections officials to a mapping between ballots and CVRs [E.K. Rescorla, personal communication, 2011; R.L. Rivest, personal communication, 2009; D. Wallach, personal communication, 2010; see also Adida [2006]]. However, we believe SOBA is the first method that requires only one commitment and that uses a risk-limiting audit to check whether the mapping is accurate enough to determine the correct winner.

We have said little about the requirement for a compliance audit. In part, this is a definitional issue: Even if the audit trail is known to have been compromised, it is our understanding that in many states, a full hand count of the audit trail would still be the “correct” outcome, as a matter of law. Hence, an audit to assess whether the audit trail was protected and preserved adequately for it to reflect the outcome according to how the voters cast their ballots is legally superfluous. We consider this a shortcoming of current audit and recount laws. Moreover, we doubt that any system can be P -resilient unless the election and the data it generates satisfies particular conditions. For instance, risk-limiting audits generally assume that the number of ballots cast in all in each contest is known. Such conditions should be checked.

We would advocate carrying out a compliance audit to assess whether the procedures as followed in the election give reasonable assurance that the audit trail is trustworthy—sufficiently accurate to reflect the outcome according to how voters cast their ballots—and to assess whether any other preconditions of the risk-limiting audit hold. The compliance audit should evaluate whether there is strong evidence that the chain of custody of the ballots is intact, or whether it is plausible that ballots were lost, “found,” altered, or substituted. The compliance audit should confirm the values of $\{N_c\}$ by ballot accounting: confirming that the number of ballots printed equals the number returned voted, unvoted, and

spoiled, for each ballot type.

If the election passes the compliance audit, a risk-limiting audit can then assess the accuracy of the reported result and would have a large chance of correcting the apparent outcome if it is wrong (by examining the full audit trail). But if the election fails the compliance audit—that is, if we lack strong evidence that the audit trail is reliable and that the preconditions for the risk-limiting audit are met—a P -resilient election framework should not declare any outcome at all.

For the method to be P -resilient, H must be binding and we must know $\{N_c\}$. Because the election official discloses H and the (fixed) length of the ballot identifiers, we can determine whether H is binding. For the method to be privacy-preserving, H must have the hiding property, which will depend on how the salts are chosen and how the CCVR files are organized. If the salts can be discovered, inferred, or guessed, or if observers have another way to reassemble whole-ballot CVRs from the CCVRs (for instance, if the CCVRs are in the same ballot order across contests), voter privacy can be compromised.

5 Conclusions

SOBA makes possible a personally verifiable privacy-preserving P -resilient canvass framework. It allows individuals to obtain strong firsthand³¹ evidence that apparent election outcomes either are correct in the first place, or are corrected by a risk-limiting audit before becoming final, without unnecessary compromises to privacy. After the procedure is complete, either all the outcomes are correct or an event with probability less than $1 - P$ has occurred. The published data structures allow the public to check the consistency of the apparent outcomes but do not allow whole-ballot cast vote records to be reconstructed, thereby preserving privacy. When all the apparent contest outcomes are correct, gathering the evidence that the outcomes are right typically will require exposing only a small fraction of ballots to observers, protecting privacy. But the data structures and auditing protocol ensure that if the apparent outcome of one or more of the contests is wrong, there is a large chance of a full hand count of the audit trail to set the record straight.

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³¹For multi-jurisdictional contests, it might not be possible to conduct an audit in a single place and time. If the audit step takes place in pieces in separate jurisdictions simultaneously, firsthand knowledge might be impossible; one might need to trust observers in other locations.

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