Detecting Voter Fraud in an Electronic Voting Context: An Analysis of the Unlimited Reelection Vote in Venezuela

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Abstract

Between December 2007 and February 2009, Venezuelans participated twice in constitutional referenda where the elimination of presidential term limits was one of the most salient proposals. Assuming voter preferences did not change significantly during that period, the 'repeated' character of these elections provide us with an excellent opportunity to apply forensic tools designed to detect anomalies and outliers in election returns in elections where electronic voting technologies were used. Similar tools were first applied by Myagkov et al. ([20], [21], [22], [23]) to the study of electoral fraud in Russia and Ukraine, and were effective in the isolation of potential cases of manipulation of electoral returns. The case of Venezuela is different because there exists no widespread agreement about the integrity or otherwise fraudulent nature of national elections, and because it is a nation where electronic voting technologies are used. Unless electoral fraud takes place in exactly the same manner in each election, an analysis of the 'flow of votes' between elections can be used to detect suspicious patterns in electoral returns. Although we do not find evidence of pervasive electoral fraud compared, for instance, to the Russian case, our analysis is useful to detect polling places or regions deviating considerably from the more general pattern.

1 Introduction

Having an electoral process with a high degree of integrity is important for the maintenance of a well-functioning representative democracy. All stakeholders must believe that an election was free from fraud and malfeasance in order for the regime that takes power after a contested election to have legitimacy. Elections that lack integrity often lead to internal political conflict, and to external political pressures. An example of such an election occurred in 2004 in the Ukraine, where electoral fraud produced massive protests as well as pressure by foreign governments, resulting in an eventual revote.

While examples of clear and widespread electoral fraud are relatively rare, concerns have arisen in recent years regarding the detection of fraud as new voting technologies and procedure are tested and deployed. These electoral innovations often involve relatively untested technologies and procedures, which sometimes appear to interact to produce potentially problematic outcomes. This may have been the case in Florida's 13th Congressional district election in 2006, when a very high undervote rate was observed in parts of the district in Sarasota County, where electronic voting machines were used. While recent research focuses on how the voting machines displayed the ballot for this race, considerable debate arose about whether there were problems with the electronic devices themselves (see Frisina, Herron, Honaker and Lewis [12]).

The need to assure the integrity of an electoral process has produced a variety of calls for better post-election analysis of election administration, sparking a number of new research efforts regarding official post-election auditing procedures.[1] Our research is closely related to these efforts, as we study the issue of post-election statistical detection of election anomalies and outliers. Of course, the statistical analysis of election returns and other election statistics has a long history, with scholars attempting to identify statistically the systematic factors impacting outcomes. Only recently, though, have scholars begun to apply their substantive and methodological tools to look not for the systematic explanatory factors of voting and election outcomes, but also for outliers and anomalies not accounted for by a classical statistical model.[2]

In this paper we discuss some of these new statistical tools for post-election forensics, with an application to re-

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[1] See, for example, Aslam, Popa and Rivest [4] or Hall [14], and the references therein.

[2] See Part Three of Alvarez, Hall and Hyde [2] for discussion of many of these different statistical tools.

cent elections in Venezuela. We use the Venezuelan case for a number of reasons: Recent elections have been important and concern widely-followed referenda on important constitutional and political issues; we want to apply these statistical tools to a nation where electronic voting is used; finally, while there have been studies of election fraud in a number of Latin American nations, the techniques we present here have not been applied to Latin American cases.

This essay proceeds as follows. In the next section we discuss briefly the research literature on election fraud, and the cross-cutting research literature on electronic voting. We then turn to a discussion of the Venezuelan case, as well as a general presentation of post-election statistical forensic analyses. Thereafter we examine the results from recent Venezuelan elections, and conclude with a discussion of the merits of these post-election statistical forensic tools for fraud detection and routine auditing of election administration practices.

2 Election Fraud and Electronic Voting

Despite the frequent use of the term "election fraud", there is little guidance in the research literature about exactly what constitutes "election fraud." For example, Alvarez, Hall and Hyde [2] note that notions of fraudulent electoral practices vary across time and political jurisdictions, and that behaviors that in one nation at a particular time are thought of as fraudulent may be perfectly legal and acceptable in other nations at other points in time. The United States Election Assistance Commission (USEAC) recently recognized this problem in a report on "election fraud", noting that it has"learned that these terms mean many things to many different people" ([29], page 11); as a consequence, the USEAC used the term "election crimes" in their study instead of "election fraud."

Regardless of how one defines "election fraud" across nations or time, there are clear cases where deliberate and illegal electoral manipulation have been observed. Latin America, our region of focus, has experienced allegations of election fraud in the past, and there has been substantial research on these allegations in some Latin American cases. Two Latin American nations in particular have been widely studied. One is Costa Rica, which was examined closely by Lehoucq and Molina [17]. Looking across nearly fifty years of Costa Rican history (made possible in part by that country's longstanding democracy), they found that election fraud varied depending on political competition, political institutions, and sociological trends. Another Latin American case that has seen substantial research is Mexico, where during the PRI's period of one-party rule, that party's tactics included the manipulation of voter registries, multiple voting, ballot box stuffing and other forms of voter intimidation (Lawson [18]). Others have studied the transition from PRI-dominance to competitive elections, with a focus on how political institutions like electoral courts can help mitigate the manipulation of elections (Eisenstadt [10]). Predominantly, though, research on election fraud in Latin America focuses on systems employing "traditional" technologies, usually paper-based voting registration and balloting systems. As the examples from Costa Rica and Mexico demonstrate, paper-based registration and ballot systems are not fraud-proof, instead they have many different types of vulnerabilities that have been exploited in past elections.

Missing from the literature are studies of the potential for election fraud that arise from the use of electronic voter registration and balloting systems (e.g., Alvarez and Hall [1]). In the United States, a widespread movement toward the greater use of electronic voting following the 2000 presidential election has proceed with fits and starts, and at this moment in time has stalled or even reversed itself (Alvarez and Hall [1]). Much of the debate about electronic voting systems in the United States concerns security and reliability; specifically, allegations that malicious tampering with the software and hardware is possible and difficult to detect (e.g., Kohno et al. [16]). These allegations have been the focus of substantial popular press, and a growing body of research (State of Alaska Division of Elections [27],[28]; California Secretary of State [7]; Ohio Secretary of State [24]).

The electronic voting system currently used in Venezuela was first introduced for the 2004 presidential recall referendum, and has since been employed in five national elections — three constitutional referenda, one presidential election, and one parliamentary election. In addition, it is used in regional and municipal races. The touch-screen system is manufactured by Smartmatic, and is an example of an electronic voting device commonly known as a "direct-recording electronic" (DRE) voting machine. The Carter Center [8] issued an observation report of the 2006 presidential election that contains a great deal of technical discussion of the electronic voting system used by Venezuela, and we draw heavily from that report as it provides one of the best English-language sources about the technical details of the voting system in Venezuela.

The Carter Center [8] analysis covers in detail many of the security features of the Smartmatic voting devices; in fact, they devote an entire section of their report to a discussion of the security features of these voting systems. Some of those features include encryption of voting information, randomization of information to deter the reconstruction of the sequence of voting, disabling unnecessary physical access ports, chain-of-custody procedures, and

the security of the system's paper receipts. Overall, the Carter Center report positively evaluated the voting system security, though it did recommend stronger physical security and chain-of-custody measures for future elections. The report specifically concluded that "The system used in Venezuela needs to be secured (and audited) it its entirety" ([8], page 29). However, following the 2006 presidential election, the Carter Center report noted a number of issues associated with the audit and it's procedures ([8], pages 40-42).

Thus, the research reported in the sections that follow offers another set of post-election tools for election officials, stakeholders, and other interested parties to use, and that thereby expands the more limited post-election audit procedures employed thus far in recent Venezuelan national elections. A post-election audit of only a small random sample of voting machines from polling stations may uncover some problems with the administration or technology used in an election; the post-election forensic tools we employ here are complementary to the random audit, as they can help discover other anomalies or problems with the conduct of a particular election. The advantage of our tools, on the other hand, is that they are designed to operate over the complete set of election returns.

3 Previous Research

The forensic tools applied here to detect election fraud using aggregate official election statistics were initially developed to study election fraud in post-Soviet states, notably Russia and Ukraine (Myagkov, Ordeshook and Shakin [21], [22], [23]). Briefly, those tools seek to discover the anomalies in the data occasioned by explicit forms of fraud, such as ballot box stuffing and falsified election protocols that introduce specific types of heterogeneity into the data. Indeed, the search for 'artificially' created heterogeneity both defines our methodology and establishes the prerequisites for its application.

We know that election returns from any national election exhibits their own natural forms of heterogeneity. Both turnout and a candidate or party's support will, generally and quite naturally, vary with demographic parameters such as urbanization, income levels, ethnic composition and even simply region. Our tools, then, seek to control for these parameters and then to look for other, potentially suspicious, sources. Of course, whether we deem these other sources suspicious depends on what we know about the election at hand. For instance, does the election involve a 'favorite son' who can be expected to wholly legitimately garner special support in some regions but not in others? Did the election concern an issue that had previously been unimportant — an issue that divided the electorate differently than before? Naturally, a concern with such things requires that in the search for suspicious heterogeneity, we must also look across elections so that we can answer questions such as 'Did a pattern suddenly emerge in the data that cannot be explained by demographic shifts?'

In contending with these substantive issues along with the various methodological issues associated with the treatment of aggregate data, Russia and Ukraine proved a useful laboratory for the development and testing of our forensic tools. Briefly, in Russia, Myagkov, Ordeshook and Shakin [23] had reasonably good priors as to where specifically election fraud was most likely to arise — notably in its ethnic republics, such as Tatarstan, Bahkortostan, Dagestan and Ingueshtia. Each region is controlled by political bosses (republic presidents) who generally win elections with numbers mimicking a Soviet past; both turnout and support for the incumbent in excess of ninety percent. In addition, it is not unusual to find subregions (rayons) in which the majority of precincts report 100 percent turnout with 100 percent of the vote going to the regional or national incumbent or to the Kremlin sanctioned party. Data from these regions should readily signal fraud and should distinguish themselves from data from other regions.

Ukraine, in turn, offered Myagkov et al. [23] nearly the perfect social science experiment in 2004 from the perspective of testing their forensic tools. Following an inconclusive first round presidential ballot in October, the November runoff pitted two candidates, Viktor Yushchenko and Viktor Yanukovich, with sharply differing ideologies and bases of support. Yushchenko's support, with his pro-NATO and reformist agenda, came largely from Western Ukraine, whereas Yanukovich's support cam from the more industrialized and largely pro-Russian East (along with Crimea). Yanukovich also received the strong support of the incumbent regime along with Russia's Vladimir Putin.

The second November runoff election, however, was marred by massive and readily documented irregularities — irregularities of such an extent that, in addition to a massive uprising among the population termed 'The Orange Revolution' that saw upwards of a half million Ukrainians permanently camped out in protest in Kyiv's central square, Ukraine's Supreme Court invalidated the result and called for a new runoff in December. As a consequence of the Court's decision and the self-evident nature of the fraud (along with, doubtlessly, behind the scenes diplomatic maneuvering), Yanukovich lost the endorsement of then president Kuchma (thereby signaling to regional political bosses that 'special efforts' on Yanukovich's behalf were no longer necessary), and several thousand election observers from Ukraine's diaspora (notably, from Canada and the United States) poured into

the country to monitor the re-runoff. Ukraine, then, presented the authors with a situation in which they had two elections, one month apart, contested by the same two candidates, with the same issues, among the same electorate but with considerably fewer opportunities and incentives for fraud in the second runoff. And since they had good priors as to where fraud was most blatant in the November runoff, they could test their indicators with the expectation that those indicators would signal fraud in one case but not in the other and that the measures they provided of the magnitude of fraud should correspond to the estimates offered by a variety of objective observers and journalistic accounts. Suffice it to say, those indicators performed as designed and suggested the presence of between 2 and 3 million 'suspect' votes in the November 2004 Ukrainian presidential runoff.[3]

4 The Case of Venezuela

We started the Venezuela analysis without strong priors about the types and location of potential election anomalies or potential fraud, or even expectations about whether there were problems in recent elections. The electoral context in this country is considerably different from that described in the Russian and Ukrainian cases. First, in the three most recent national elections, there are few polling places with 100 percent turnout where the official position, or any other alternative, wins close to 100 percent of the vote. Even in regions with strong support for the government, the opposition wins a 'reasonable' positive share of the vote. Second, international observers such as the European Union Election Observation Mission (EUEOM) positively evaluated recent electoral processes. For example, in their report on the 2006 presidential contest, the EUEOM congratulated the National Elections Council (CNE), as well political actors and social movements, for "creating conditions to hold elections that are acceptable to all stakeholders" and argued that "the electronic voting system established in Venezuela is efficient, secure, and auditable" (EUEOM [11], page 2). Election observers pointed to minor irregularities and suggested improvements to the electoral system, but they did not raise serious allegations of electoral malfeasance. Although local organizations — such as SUMATE, as well as the opposition — made great efforts to monitor and report incidents of fraud, there exists no proof of widespread irregularities in recent elections. The most notable case of an election where Chavez's position won a disproportionate share of the vote is the 2005 parliamentary contest, where the Movimiento V Repblica (MVR) obtained 116 out of 167 seats in Venezuela's National Assembly. This lopsided victory occurred, however, because the opposition boycotted the election, alleging there were not enough guarantees for a free and fair vote. Unsurprisingly, the opposition's participation and vote share was small and overall turnout only reached 25.3 percent.

Some of the main opposition arguments about the non-democratic character of the government relate to the constitutionality of the articles included in the 2007 and 2009 constitutional referenda. The 2007 referendum consisted of, among other things, the removal of presidential term limits, abolishing the autonomy of the central bank, granting the president control over international currency reserves, expropriation of large land estates, and reducing the work day from eight to six hours. People opposed to Chavez's 'reforms' argued this last provision was introduced merely as a sweetener to attract votes. In December 2007, citizens voted on two bundles of constitutional changes, one of them (bundle A) containing all the reforms originally proposed by Chavez, including the unlimited reelection proposal. Paradoxically, it was the opposition boycott of the 2005 election that gave Chavez an overwhelming majority in the National Assembly and allowed Chavez to readily include his favored proposals in the 2007 referendum. Still, Chavez's position lost by a small margin, which shows that the Venezuelan case differs significantly from the Russian or Ukrainian cases previously summarized.

After the 2007 defeat, Chavez accepted the outcome but publicly warned that this was just a temporary defeat — he said "no pudimos, por ahora", meaning "for the moment, we couldn't" making allusion to the phrase he used when surrendering after the 1992 attempted *coup d'tat*. After the referendum, banners appeared throughout Caracas showing the phrase "por ahora" ("for the moment"). Moreover, while the constitution does not allow a similar constitutional reform to be put to a vote twice during the same National Assembly period, Chavez came up with a new proposal which excluded most of the 2007 reforms, but still included the unlimited reelection proposal, together with the elimination of reelection terms for other public offices, and managed to get approval from the Constitutional Court for holding a new referendum in February 2009. In this context, it is reasonable to wonder to which extent the integrity of the electoral process would be preserved in 2009.

The data about electoral returns we use here to address this issue comes from two sources. First, we downloaded the 2007 and 2009 referendum results from the CNE webpage using a Python script to store the data in a spreadsheet. Later, we found it convenient to compare the referendum outcomes with those from the 2006 presidential election. The data corresponding to this last race was downloaded from the ESDATA website, which contains

[3] Similar tools were applied to the 2004 Russian presidential election, 2007 Russian parliamentary election, and 2007 Ukrainian election (see Myagkov et al. [22], [23]).

spreadsheets with official election results at the lowest level of aggregation available.[4] In addition, in different sections of our analysis we classify polling places, or control for socio-demographic differences, using census information. This data was downloaded from the web page of the National Statistics Institute (INE).[5]

5 Methodology

Briefly, our forensic indicators fall into three categories. The first looks for anomalies in the distribution of turnout and the relationship between turnout and a candidate or party's reported share of the eligible electorate, the second looks for anomalies in the patterns of the numbers — the digits — employed in official protocols, and the last category applies various econometric techniques to estimate the flow of votes between elections.

With respect to the first category, suppose we in fact have a homogeneous data set (e.g., polling stations or precincts) in the sense that turnout varies within it for purely random reasons, for reasons having little if anything to do with demographic parameters or with those factors that impact a candidate or party's level of support. In this case the overall distribution of turnout across the data should be approximately normally (and in particular, unimodally) distributed, with some districts reporting high turnout, some reporting low turnout, but the overwhelming majority reporting turnout rates at or near the average. Now, however, suppose we introduce a specific form of heterogeneity into the data by literally stuffing the ballot boxes of a subset of precincts with votes for a specific candidate. The turnout distribution of that subset will be moved to the right so as to create an elongated tail for the overall distribution. And if we stuff enough ballots into those boxes, we can actually create an overall bimodal distribution. Bimodal distributions of turnout, then, are one of the 'red flags' we can use when looking for fraudulently cast votes.

Next, suppose we look at the relationship between turnout and a candidate's share of the eligible electorate. Again, if turnout is unrelated to a party's relative support, that relationship (i.e., the slope of the line relating turnout, T, to share of the eligible electorate, V/E) should approximately equal the party's overall share of the vote in our sample of the data. In other words, suppose turnout increases by 100 people. Then if turnout is unrelated to the party's support relative to the opposition, that party should enjoy a share of those 100 additional voters equal approximately to its general share of the vote. Surely it should not gain more than 100 votes, nor should it experience any loss in votes. Thus, regression estimates of the relationship between V/E and T in homogeneous data should fall in the interval [0,1], so once again, regression estimates outside of this interval serve as another indicator of potentially fraudulently reported votes.

The second category of forensic indicators examines patterns in digits in official election returns (Berber and Scacco [5], Shpilkin [26]).[6] Suppose for example that election protocols are filled out with little or no regard for ballots actually cast. And suppose, moreover, as is actually the case in places such as Russia, that there are few penalties for committing fraud, provided only that the fraud benefits the incumbent regime. In this case we can readily imagine a heuristic for filling out protocols — say, officially reported turnout — in which numbers are simply rounded off to 0 or 5. Looking at the distribution of last digits, then, can serve as an additional piece of forensic evidence. But suppose those who would commit fraud attempt to be more sophisticated and deliberately avoid the over-use of 0's and 5's. So consider instead the last two digits of official tabulations. It is an experimentally verified fact that if we ask people to write sequences of random numbers, they will tend to write down paired numbers (e.g., 2 2 or 3 3 or 7 7) less frequently than we would actually expect in a purely random sequence (Chapanis [9], Rath [25], Boland and Hutchinson [6]). A fourth 'red flag', then, is a distribution of last and next-to-last digits that departs significantly from what we would expect from a purely random process.

An additional forensic indicator entails estimating the 'flow of votes' from one election to the next. Here, of course, we are attempting to estimate where the votes of one party or candidate (or position on a referendum) went to in successive elections with the expectation that no party or candidate should win more than 100% of the vote of some earlier candidate or party. Needless to say, treating aggregate data so as to obtain estimates of this sort encounter any number of problems associated with ecological correlation. Nevertheless, a variety of econometric techniques have been developed for treating such issues, and their application constitutes our final forensic indicator. Specifically, we estimate Goodman regressions of the following form:

$$y_{1i} = \beta_{11}x_{1i} + \beta_{12}x_{2i} + \beta_{13}(1 - x_{1i} - x_{2i}) \quad (1)$$

$$y_{2i} = \beta_{21}x_{1i} + \beta_{22}x_{2i} + \beta_{23}(1 - x_{1i} - x_{2i}) \quad (2)$$

[4]The ESDATA web site is http://www.esdata.info/venezuela. To check the accuracy of the data available in this location, we compared their 2009 referendum database, with the one we constructed with the data downloaded from the CNE, and results were 100% consistent.

[5]The INE webpage is www.ine.gov.ve, and the section we downloaded the data from is called "Síntesis Estadística Estatal 2008."

[6]There are other scholars who also study patterns in digits to detect electoral fraud. For instance, Mebane [19] looks for deviations from the 'second-digit Benford's law'. In this paper, we follow a different approach, by searching for patterns of non-random behavior in the distribution of the last-two digits.

$$y_{3i} = \beta_{31}x_{1i} + \beta_{32}x_{2i} + \beta_{33}(1 - x_{1i} - x_{2i}) \quad (3)$$

where each y_i represents a share of the eligible electorate we want to explain - *yes*, *no* and abstention proportions in the case of referenda, and the x_i's indicate the proportions of people voting for the government's favorite alternative, for the opposition's favorite alternative, or abstaining in a previous election. Each β coefficient represents the proportion of a previous vote share that 'flows' to each alternative in the most recent election. Therefore, these coefficients should lie between 0 and 1. Further, assuming no change in the electorate, any voter must either abstain, or vote for one of the available alternatives. Thus, coefficients lying in a similar column — i.e., those associated with the same x_i, should add to one across the three equations. If this holds, the nine β coefficients should add to three, predicted y_i's should lie between zero and one, and the sum of the predicted y_i's should add to one.

Still, since we do not use individual level data, aggregation bias may lead to estimated coefficients that are out of bounds. The usual solution applied in the literature has been to use an estimation method restricting coefficients to lie within 'theoretically prescribed' bounds (King [15]). Myagkov et al. [23], however, argue that electoral fraud may lead to negative or above-one coefficients even in the absence of aggregation issues, so restricting the coefficients is not desirable because it does not allow us to detect cases of electoral fraud. Still, to correctly assess the cause of an out-of-bounds coefficient, we need to take steps to mitigate the ecological inference problem. Myagkov et al. [23] use a semi-parametric procedure, which consists of estimating the model within clusters of similar regions, and then constructing a weighted average of the coefficients to obtain the overall results. In this paper, we employ a method with a similar aim. Specifically, we estimate Goodman regressions with random effects, where coefficients are allowed to vary by region, or to vary by levels of census variables.[7] This procedure can be thought of as the intermediate step between a complete pooling approach where the model is estimated at the national level without random effects, and a no-pooling approach where the model is estimated independently within each region.

Finally, an additional method we use to detect anomalies in electoral results follows a test done by Alvarez and Katz [3]. The authors first fit a model to the 1998 governor and senate election results in the state of Georgia, using the results of the 1996 presidential election as explanatory variables. Then, they use the estimated coefficients to predict the 2002 election results. In that way, they seek to detect if outcomes deviate significantly from expectations. In the case of Venezuela we first fit the *yes* share of the vote in the 2007 referendum, using the proportions of the eligible electorate abstaining or supporting Chavez and Rosales in the 2006 presidential election as explanatory variables — controlling for a small set of socio-demographic variables, and then use the estimated coefficients and 2007 electoral results to predict the *yes* share of the vote in the 2009 referendum. The idea is to check whether a subset of the polling places deviates considerably from our forecast.

Of course, if fraud occurs in both elections in precisely the same way and in the same localities, this method will not detect anomalies. This fact, though, merely serves to emphasize that our forensic indicators, taken together or separately, are precisely that — indicators. None of them, separately or together, can be said to 'prove' the existence or absence of fraud. Much as in a criminal investigation, they merely provide evidence, out of which we, as analysts, most form a coherent view of an election — a view that is consistent with that evidence. There is no black box into which we plug official election returns and out of which comes a determination as to whether an election was or was not tainted by significant fraud. Our indicators provide us with alternative ways of looking at official election returns and must, necessarily, be combined with some substantive expertise about the society and election at hand. Only the substantive expert can answer such questions as: Were adequate controls in place for heterogeneity in the population; are there alternative explanations for why, for instance, increases in turnout benefited only one candidate or party; did fraud take more subtle forms so as to be undetectable by our indicators; did issues arise in one election but not in others that give that election a distinctly different character; or did one candidate or another enjoy administrative advantages that occasion suspicious patterns in the data but that cannot be classified as illegal fraud? Only after these questions are asked and answered can we begin drawing definitive conclusions from our forensic indicators.

6 Results

Most of the analysis done for individual elections was done using 'mesa' (table) level data. However, since the number of tables changes within one same polling place from election to election, the cross-election analysis was done comparing polling-place level results. Specifically, we used polling place IDs — constant across elections, to exactly match 8,815 polling places which were open during the three elections of interest. The average number of voters and eligible voters per polling place in 2009 was 1,230 and 1,744, respectively.

It was very important for our analysis to count with 2007 turnout figures. Still, this information is not available at the CNE's web site. To solve this issue, we esti-

[7] Estimations were done by MLE, using the lme4 R package.

mated polling place level registration numbers for December 2007 by interpolating between the December 2006 and February 2009 official registration figures for each polling place. Turnout rates were computed by taking the total number of votes — including both valid and invalid votes, as a percentage of the total number of registered voters. According to the news reports, turnout in the 2007 constitutional referendum was approximately 56 percent. Based on our own estimations, turnout in that election situated at 58 percent — with average and median figures of 57 percent and 59 percent, respectively. These figures are substantively smaller than the 75 percent and 70 percent officially reported for the 2006 presidential and 2009 constitutional referendums, respectively — or 74 percent and 69 percent in our sample of matched polling places. Figure 1 compares the distribution of turnout across the three elections. It shows that turnout was largest and had a relatively low variance in 2006, then it decreased considerably in 2007, and finally increased in 2009. The only slight anomaly we observed in this completely pooled data is some thickness in the right tail of the 2006 turnout distribution.

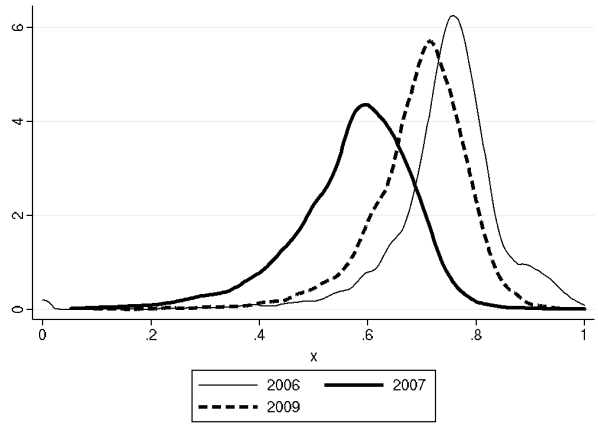

Figure 1: Distribution of Turnout in Recent Elections

The official result of the 2006 presidential election was a 64.8 percent Chavez victory, relative to the total number of valid votes. If instead we consider the proportion of the eligible electorate, Chavez vote share was equal to 45.6 percent. Then, in the 2007 referendum, the *no* option won the election corresponding to both bundles of constitutional articles. Specifically, the *no* vote share stood at 50.7 percent in the case of bundle A. If we consider the proportion of the eligible electorate, the vote share received by the *no* option — in bundle A, was equal to 29.7 percent. Last, the *yes* option won the 2009 referendum with a 54.9 percent share of the vote — or 37.4 percent in terms of the eligible electorate. Figure 2 shows the results of the 2006 presidential election, and the 2007 and 2009 constitutional referendums, using ternary plots. The upper vertices correspond to abstention, the lower left vertices correspond to the alternative supported by the *Chavismo*, and the lower right vertices correspond to the alternative supported by the opposition. Any point within these triangles indicates an election outcome — where the proportions associated with the different alternatives add to 1. To interpret these figures, notice that standing at any vertex, lines parallel to the opposite side represent similar vote shares associated with the vertex's label. Also, vote shares decrease linearly as we move away from the vertex. Since the centers of the triangles are equidistant from all extremes, they represent a situation with the exact same proportion of votes for each alternative.[8] These figures show that while the 2009 distribution of outcomes is very similar in shape to the one observed in the 2006 presidential election, the 2007 constitutional referendum outcome looks like a special case of relatively low turnout.

Figure 3 shows the distribution of turnout by state. In general, state-level distributions follow the national pattern observed in Figure 1. The most noticeable deviations from the normal distribution occur in the 2006 presidential election — consistent with the thick right tail observed in the distribution of national turnout, with some states exhibiting relatively large second modes to the right — in particular, this happens in Cojedes and Vargas. This last state shows slightly multimodal distributions in all three elections. Then, in 2007, there is a set of states whose turnout distribution does not look normal, but slightly skewed and with thick tails — we refer to Bolivar, Delta Amacuro, Guarico and Zulia. Finally, in the case of the 2009 referendum, most densities look unimodal and tightly clustered around some point, and are not skewed to any side — although Barinas and Taćhira exhibit small hills to the left, and Vargas to the right.

As mentioned in the previous section, both alternatives should benefit partially from an increase in turnout — or, at least, none should get hurt. A failure of this hypothesis raises a 'red flag' about artificially inflated turnout favoring one of the alternatives. To test it we plot the relationship between turnout and the shares of the vote for each alternative, by state, for sufficiently similar polling places. The criteria for constructing subsets of similar polling places by state, were splitting the data in two, depending on whether the *yes* option won the previous election in that polling place (see Figure 4), or whether the opposite outcome took place (see Figure 5). In the case of those regions where the *yes* won in 2007, we observe slightly negative slopes in the *no* shares in the states of Cojedes and Delta Amacuro. Also, in those regions where

[8] Actually, the abstention percentage represented in the ternary plots includes both actual abstention as well as invalid vote. In the case of the 2006 presidential election and 2009 constitutional referendum, invalid votes stood at 1.0% and 1.2%, respectively.

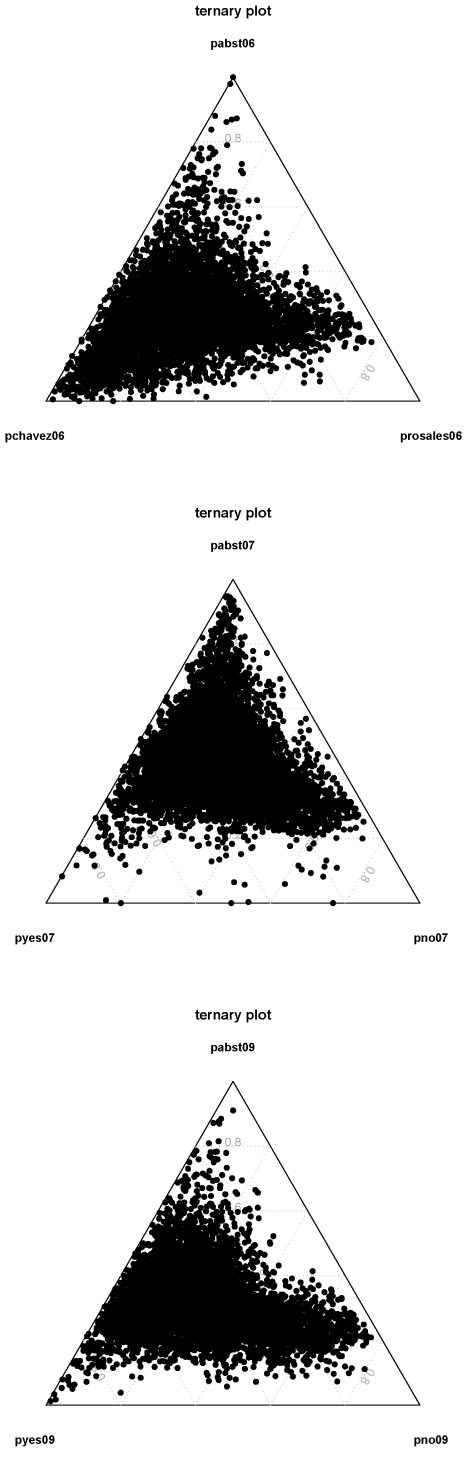

Figure 2: Recent Election Outcomes

the *no* won in 2007, we observe negative *yes* slopes in several states — Bolivar, Carabobo, Cojedes, Lara, Merida, Portuguesa, Trujillo and Yaracuy. The most extreme cases are Carabobo and Cojedes, where the *yes* share falls considerably as turnout increases. In general, observed figures for the 2009 election are mostly consistent with our hypothesis. A similar analysis of the 2007 election results is also consistent with the hypothesis that both shares of the eligible electorate benefit partially from an increase in turnout. However, we do observe deviations from our expectation in some states, which warns about potential electoral fraud in some of the regions under consideration. Since our hypothesis rests on the assumption of homogeneity across observations — i.e. observations which only differ in turnout, deviations may also be due to heterogeneity not accounted for by our bivariate regressions and scatter plots.

Next, we proceeded to analyze the patterns of digits of the 'mesa' (table) level electoral results. Figure 6 shows histograms with the density of the last digit for each alternative in the three elections of interest — the 2006 presidential election, the 2007 constitutional referendum (bundle A), and the 2009 constitutional referendum. In addition, we also include the density of the last digit corresponding to the outcomes of bundle B of the 2007 constitutional referendum. We do not observe any obvious non-random pattern — such as 0/5 rounding, or avoidance of zeros and fives. All last digits have approximately the same 10 percent incidence. Also, Figure 7 shows matrix plots of the density of the last two digits. Each cell corresponds to a combination of a next-to-last (row) digit, and a last (column) digit, with darker tones indicating larger incidence. If individuals unconsciously avoid paired numbers such as 2 2, 3 3, and so on, then we should observe white diagonals. However, this pattern does not show up in any occasion — and neither does the opposite pattern where people focus on 2 2, 3 3, and so on. In general, none of the matrices shows an obvious non-random pattern in the distribution of last two digits. A shortcoming of this tool is that electronic fraud may be designed in such a way that manipulation of the last digits is completely random. If this were the case, this particular indicator would not be useful to detect fraud.

After that, we turn to the analysis of the flow of votes between different elections. Figure 8 shows the proportion of the *yes* vote in the eligible electorate in 2009, as a function of the same variable in 2007. We observe that the proportion of support for the *yes* increases in most polling places, especially where the *yes* vote was small in 2007. Even though most polling places are clustered around the regression line, some observations are far off. Since deviations may be due to the aggregations of results from dissimilar regions, we built Figure 9 showing comparable graphs by state. Again, state-level results show

patterns similar to those observed at the national level, though outcomes are more tightly clustered around the fitted regression line. Some states, such as Aragua, Delta Amacuro, and Distrito Capital exhibit small clusters of polling places where the proportion of *yes* votes increased considerably in 2009 relative to most voting centers.

To further learn about the flow of votes between elections, we estimated a set of Goodman equations similar to the one represented by equations (1) through (3). Table 1 shows the results with complete pooling, where each equation is estimated by OLS. The model fits the data well — with each equation explaining more than 95% of the variance in the dependent variable. Still, three of the coefficients lie out of bounds. To mitigate the potential aggregation bias, we re-estimated similar equations specifying random coefficients by region. In total, we considered 366 regions — which in most states correspond to municipalities, except for the Distrito Capital and Vargas, where our regions correspond to 'parroquias.' In this case, we still find that two of the average coefficients are negative, but closer to the [0,1] interval compared to the complete pooling approach. The main result following from this table is that voters who supported the *yes* option in 2007, overwhelmingly support the *yes* option in 2009 — this is not surprising because the 2007 reform proposal was even more radical than the 2009 reform proposal. Similarly, those voters who supported the *no* alternative in 2007, overwhelmingly choose the *no* alternative in 2009. None of these results is surprising.

The most interesting result corresponds to the flow of votes of from those who abstained in 2007. Suppose all of the increase in turnout observed between both elections was artificial participation created by the government to inflate the *yes* results. Then, the coefficient in the upper right corner should be relatively large. However, we observe a different pattern. Approximately 56 percent of those who abstained in 2007 continue to abstain in 2009. And among those who choose to participate, two thirds support the *yes* position, and one third support the *no* position. Thus, even though the *yes* alternative was the main beneficiary of the increase in turnout, the no alternative also received a fairly large proportion of the new vote. Also, the random coefficients model is useful because it allows us to detect regions deviating significantly from average estimates. Figure 10 shows that the random effects associated with some of the regions were significantly different from the average effects. For instance, in the Pedernales municipality of Delta Amacuro, the proportion of the 2007 *yes* share received by the *yes* alternative in 2009 is equal to 146 percent, very out of bounds, and way above the estimates from any of the remaining regions. Also, in the case of Acevedo municipality in the state of Miranda, the proportion of the 2007 *no* vote share received by the *no* alternative is equal to 111%, an out-of-bounds and relatively large estimate in comparison to other regions. Table 3 shows the results of a similar estimation with non-nested random effects by average age, gender and proportion of rural population, for the first equation. According to the deviance criterion — a measure of error — the model with random coefficients by region fitted the data better than this second model. However, the non-nested alternative allows us to learn about the determinants of the effects. For instance, according to panel (c) in Figure 11, the proportion of the 2009 *yes* vote originating from those abstaining or voting *yes* in 2007, decreases with the proportion of rural population, while the opposite happens with the proportion received from those voting *no* in 2007.

Table 1: Flow of Votes between 2007 and 2009
Complete pooling, OLS estimation

	Yes 07	No 07	Abst 07
Yes 09	0.99	-0.09	0.27
No 09	-0.15	1.02	0.13
Abst 09	0.14	0.05	0.58
Vertical sum	0.98	0.98	0.98

Table 2: Flow of Votes between 2007 and 2009
Random effects by region, M.L.E.

	Yes 07	No 07	Abst 07
Yes 09	0.97	-0.08	0.28
No 09	-0.11	0.94	0.14
Abst 09	0.13	0.10	0.56
Vertical sum	1.00	0.96	0.98

Table 3: Flow of Votes between 2007 and 2009
Random effects by demographic indicators, M.L.E.

	Yes 07	No 07	Abst 07
Yes 09	1.00	-0.07	0.27
No 09	-0.15	0.97	0.16
Abst 09	0.13	0.09	0.56
Vertical sum	0.98	0.99	0.99

Table 4 shows the results of a Goodman regression analysis with random coefficients by region, to explain the flow of votes between the 2006 presidential election and the 2009 constitutional referendum. The main result is that a substantial proportion (20 percent) of those who voted for Chavez in 2006 abstained in the 2009 referendum. Also, 94 percent of those who voted for Rosales in 2006 choose the *no* alternative in the 2009 constitutional referendum, and 11 percent abstain. Second, 72 percent of those who abstained in 2006, abstain again in 2009, while the rest (20 percent), support the *yes* alternative. Table 5

shows the results of a similar analysis to explain the flow of votes between the 2006 and 2007 elections. In this case, we find that 36 percent of those who voted for Chavez in 2006, decide to abstain in 2007. Also, among those voting for Rosales in 2006, 87 percent vote *no* in 2007, while 19 percent abstain. Last, 77 percent of those who abstained in 2006, also do in 2007, while 16 percent support the *yes* alternative.

Table 4: Flow of Votes between 2006 and 2009
Random effects by region, M.L.E.

	Chavez 06	Rosales 06	Abst 06
Yes 09	0.74	-0.09	0.24
No 09	0.04	0.94	0.04
Abst 09	0.20	0.11	0.72
Vertical sum	0.98	0.96	1.00

Table 5: Flow of Votes between 2006 and 2007
Random effects by region, M.L.E.

	Chavez 06	Rosales 06	Abst 06
Yes 07	0.60	-0.08	0.16
No 07	0.03	0.87	0.08
Abst 07	0.36	0.19	0.77
Vertical sum	0.99	0.98	1.01

Our final indicator corresponds to a comparison of the observed *yes* vote share in 2009, with a forecast of the *yes* vote share for 2009. Predictions were computed using the estimated coefficients of a model first fitted to the 2007 election results — with predictors including the 2006 outcome, plus a set of socio-demographic control variables, which explained 98 percent of the variance in the 2007 *yes* vote share. Figure 13 shows the result of this comparison. Most of the polling places exhibit larger than expected vote shares, but we do not observe any group of polling places located away from the rest of the data — most points are clustered around a line with slope close to one and positive intercept. The model did worst among those polling places where the *yes* position was predicted to obtain between 60 percent and 80 percent of the vote — instead, many of those locations exhibited a *yes* vote share between 80 percent and 100 percent.

7 Conclusions

In this paper, we used a set of statistical tools to detect outliers and anomalies in election returns in two recent constitutional referendums in Venezuela where electronic voting technologies were used. Studying the integrity of these elections is challenging because manipulation of the software used for electronic voting could lead to types of fraud difficult to detect with some of our forensic indicators. Still, the elimination of the presidential term limits stood out as one of the main proposals in both constitutional referendums. Assuming there were no considerable socio-demographic changes in the population between both elections, it would be suspicious if the 2009 share received by the *yes* alternative changed dramatically in polling places where the same position lost the election in 2007. Even with a well designed electronic fraud, considerable changes between both elections may be detected with a careful analysis of the flow of votes.

Our main observation is that most of the new votes received by the *yes* position came from polling places with large abstention in 2007. Still, a third of those who abstained in the previous election participated and supported the *no* position, implying that not all the increase in turnout was artificially created by the government to get the reform approved. Further, the new distribution of turnout, as well as the election outcome, was very similar to that observed in the 2006 presidential election, suggesting the 2009 election was one where turnout recovered and people voted relatively similar to how they did in 2006. Of course, if fraud took place in 2006 and it was carried out in the same way as in 2009, then we would not be able to detect it using our flow of votes analysis. But this begs the question of why would the government replicate the same type of manipulation in 2009, and not in 2007? If they were capable of successfully conducting fraud in 2006, they could also have used it to approve the radical reforms proposed in 2007.

Even though we did not find evidence of widespread electoral fraud taking place in the last referendum, we did observe anomalies and outliers in different steps of our study. For instance, the state of Delta Amacuro showed both anomalies in the turnout distribution, as well as in the relationship between vote shares and turnout. Further, a municipality in the same state was prominent for having a very large and out of bounds coefficient associated with the share of the *yes* vote in 2007 'flowing' to the *yes* vote in 2009. However, this is no proof that electoral fraud took place in Delta Amacuro. As we warned in the methodological section, our indicators do not constitute, by themselves, proof that electoral fraud did or did not take place in any particular region, or in the country as a whole. They should be interpreted by experts with knowledge about the characteristics and heterogeneities of the different areas, and employed as a complement to the information arising from pre- and post-election audits, as well as reports from election observers.

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Appendices

A Tables and Figures

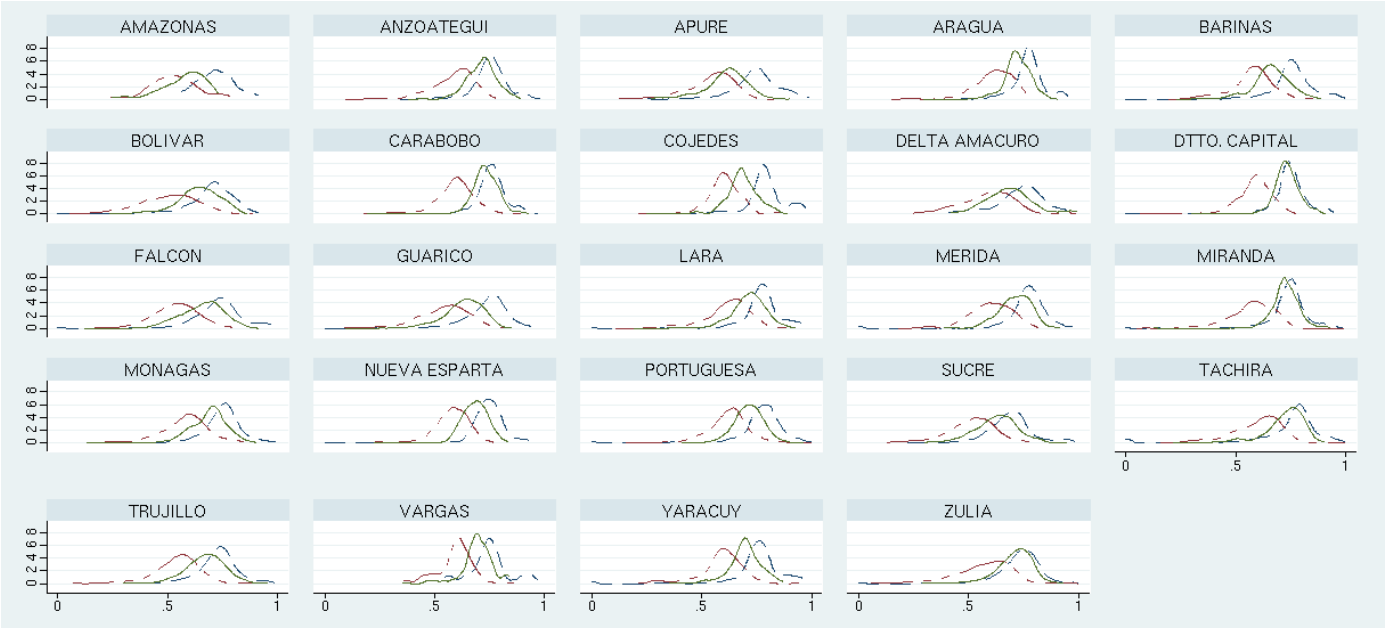


Figure 3: Distribution of rurnout in recent elections. The blue, red and green densities correspond to turnout in the 2006 presidential election, 2007 constitutional referendum and 2009 constitutional referendum, respectively.

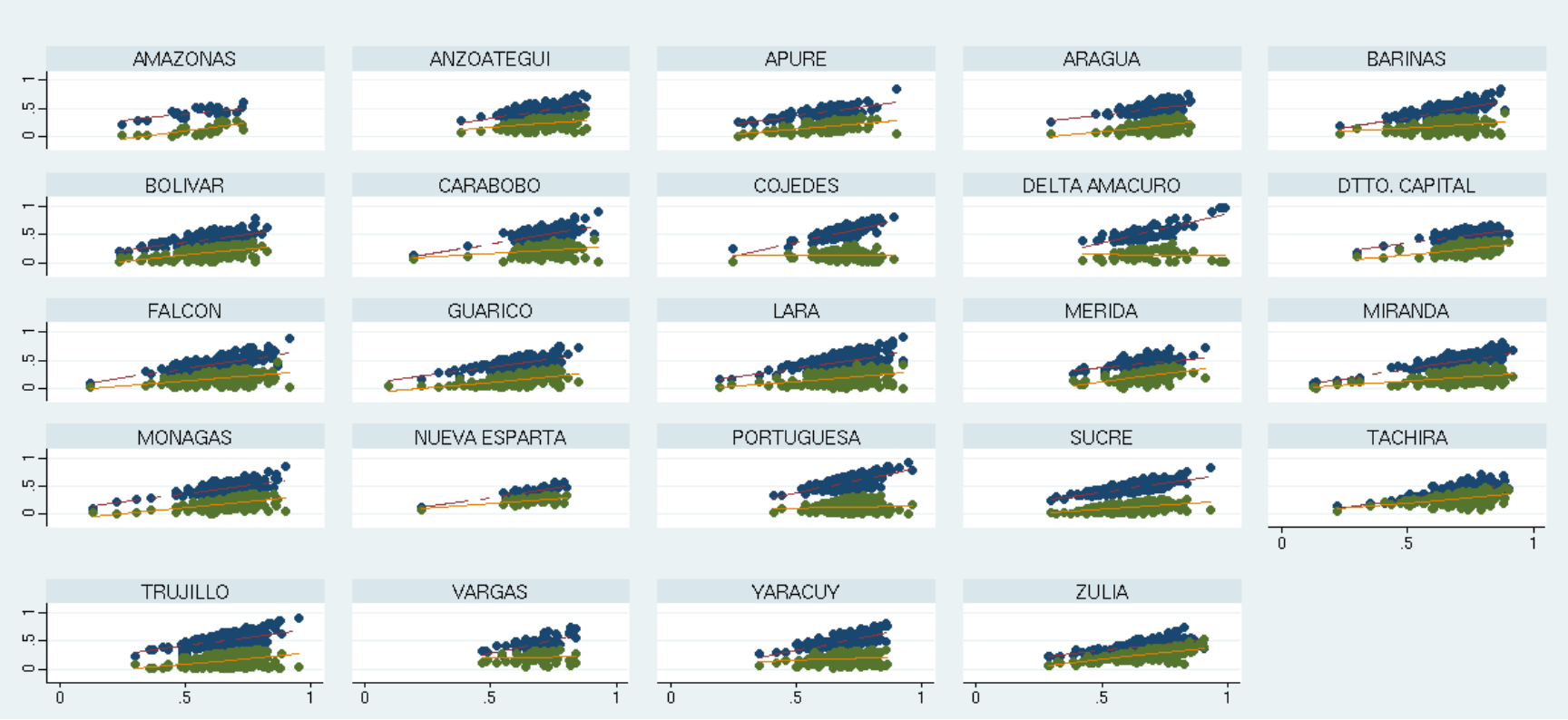


Figure 4: Relationship between turnout (horizontal axis) and vote shares of eligible electorate (vertical axis) in polling places with victory of the *yes* alternative in 2007. Blue dots indicate vote shares for the *yes* option, and green dots indicate vote shares for the *no* option. Lines correspond to fitted bivariate linear regressions between turnout and vote shares.

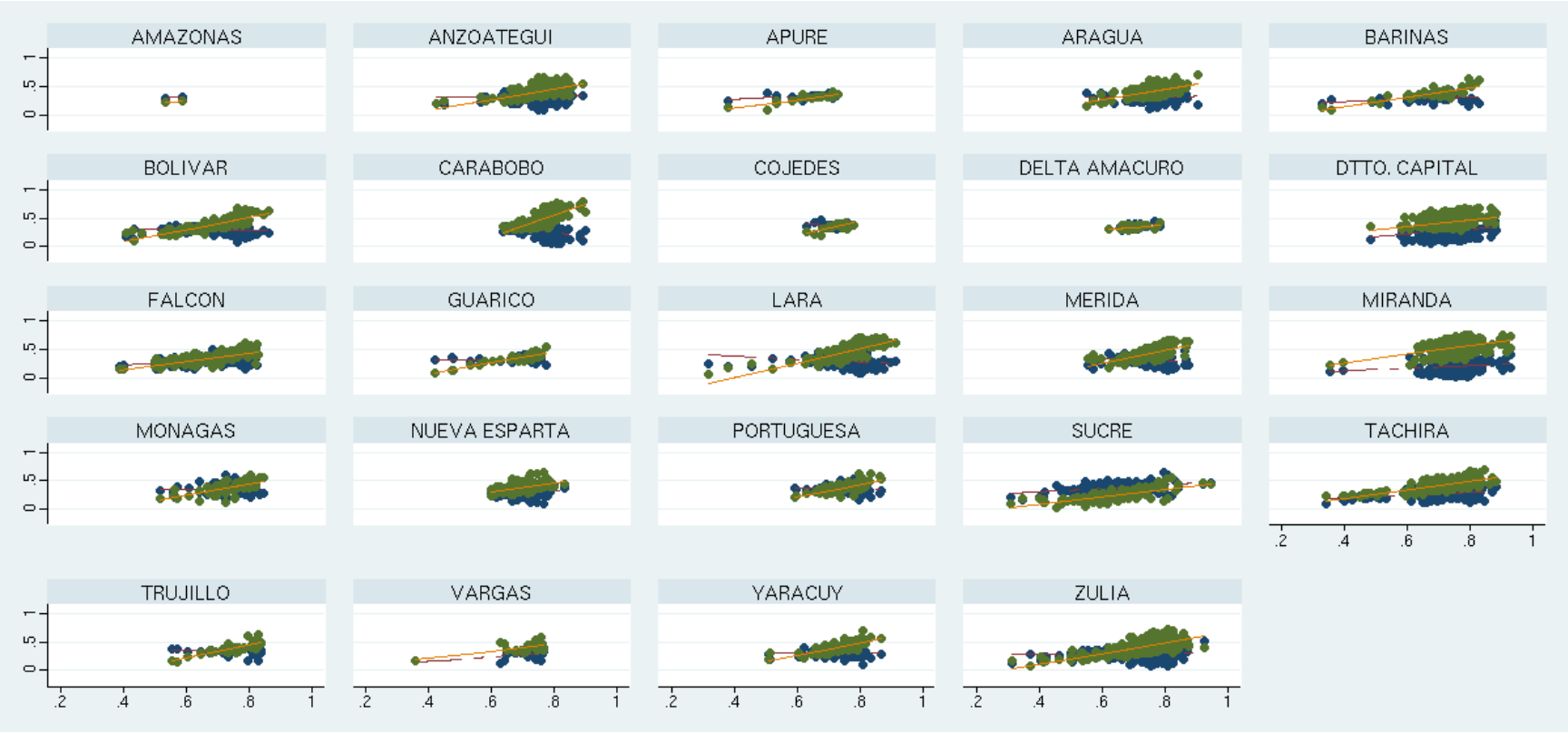


Figure 5: Relationship between turnout (horizontal axis) and vote shares of eligible electorate (vertical axis) in polling places with victory of the *no* alternative in 2007. Blue dots indicate vote shares for the *yes* option, and green dots indicate vote shares for the *no* option. Lines correspond to fitted bivariate linear regressions between turnout and vote shares.

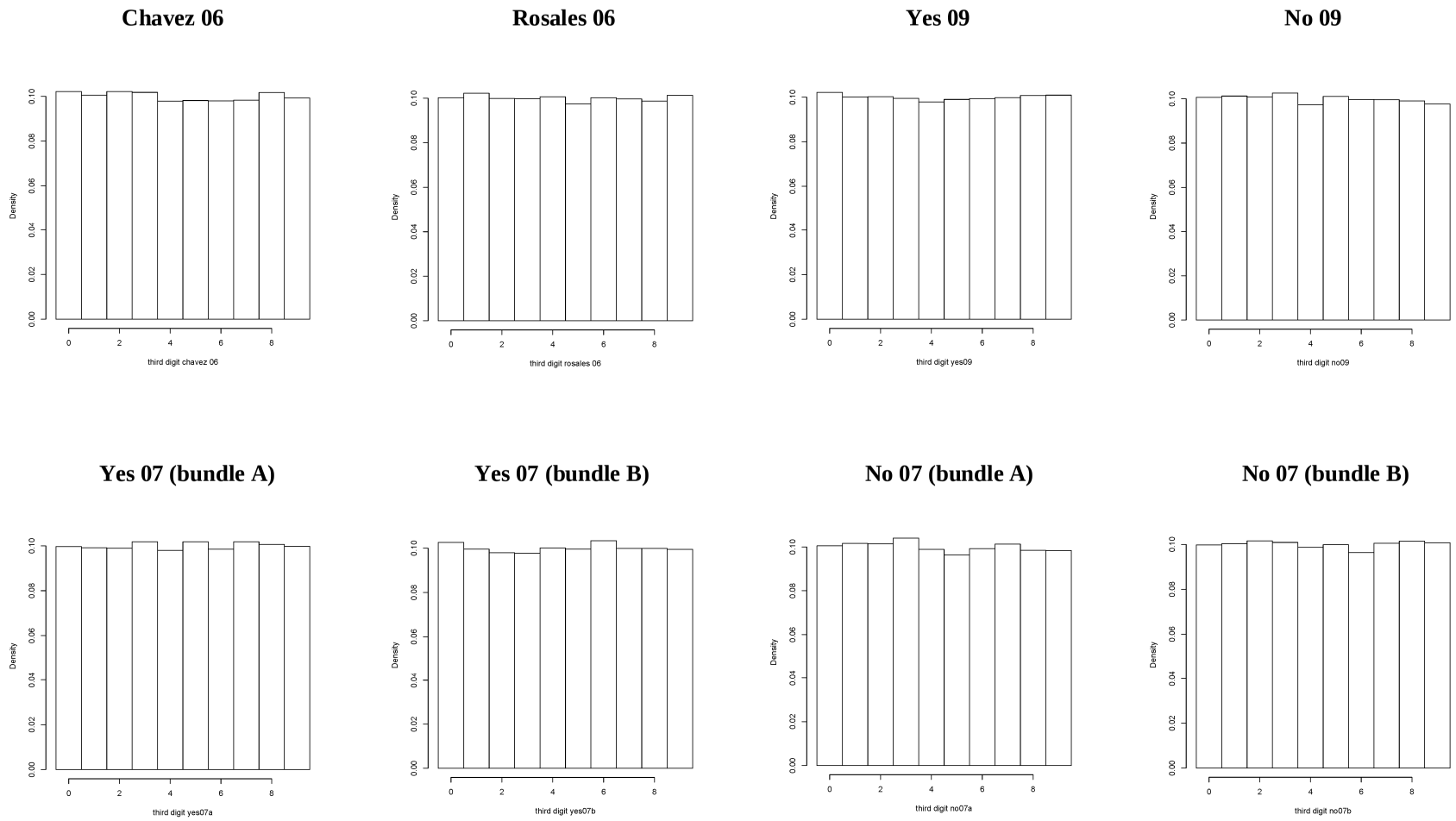


Figure 6: Distribution of the last digit

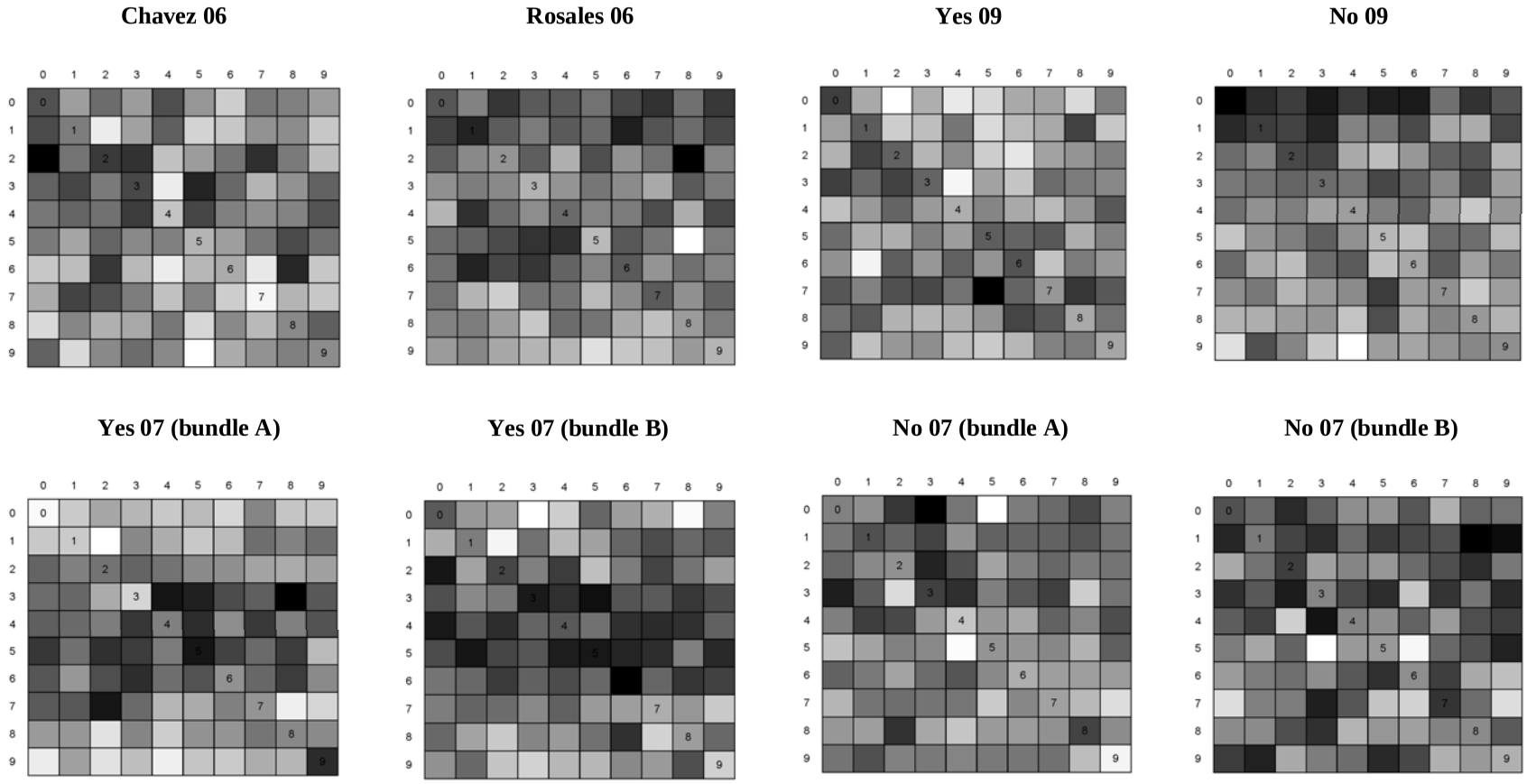


Figure 7: Distribution of last two digits. In each figure, the horizontal dimension corresponds to the last digit, and the vertical dimension corresponds to the next to last digit. Each cell represents the density of the last two digits, with darker tones indicating larger incidence.

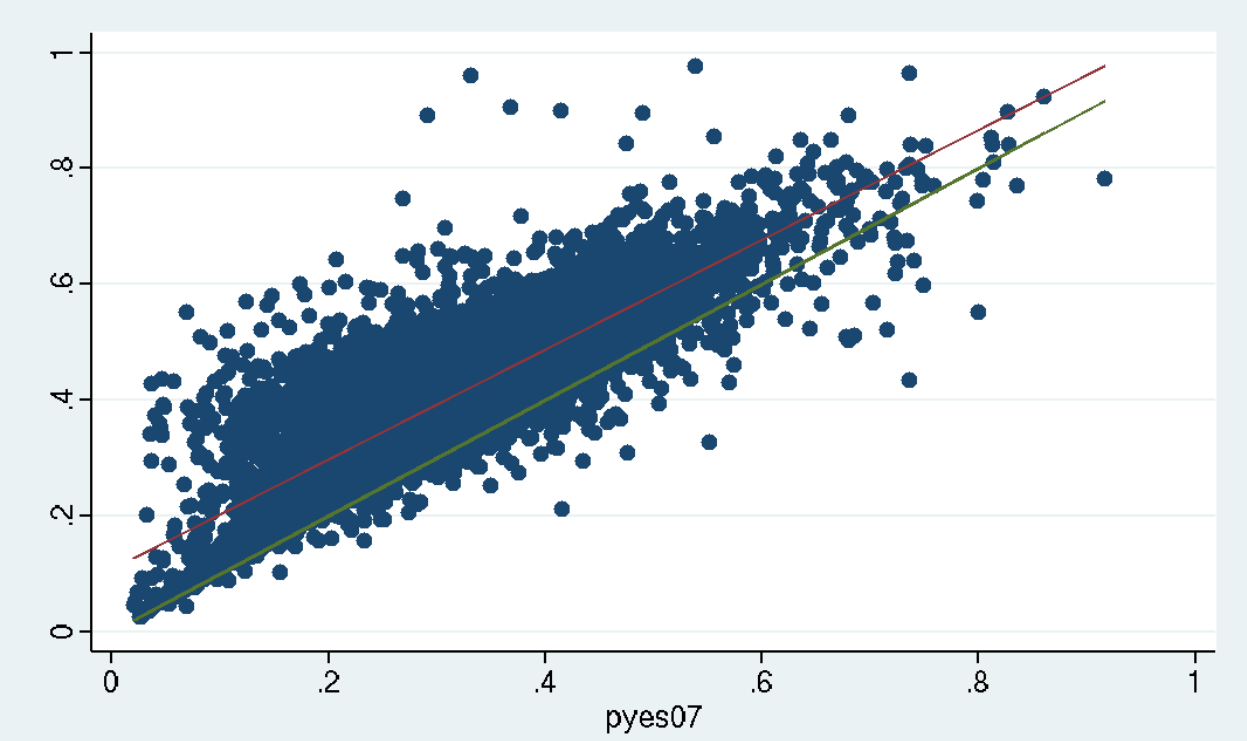


Figure 8: Relationship between *yes* shares in 2009 and *yes* shares in 2007. The red line represents the fitted results of a linear regression between the proportion of *yes* votes in the 2007 constitutional referendum (bundle A), and the proportion of *yes* votes in the 2009 constitutional referendum. The green line represents a situation where the proportion of *yes* votes in both elections is the same (45° line).

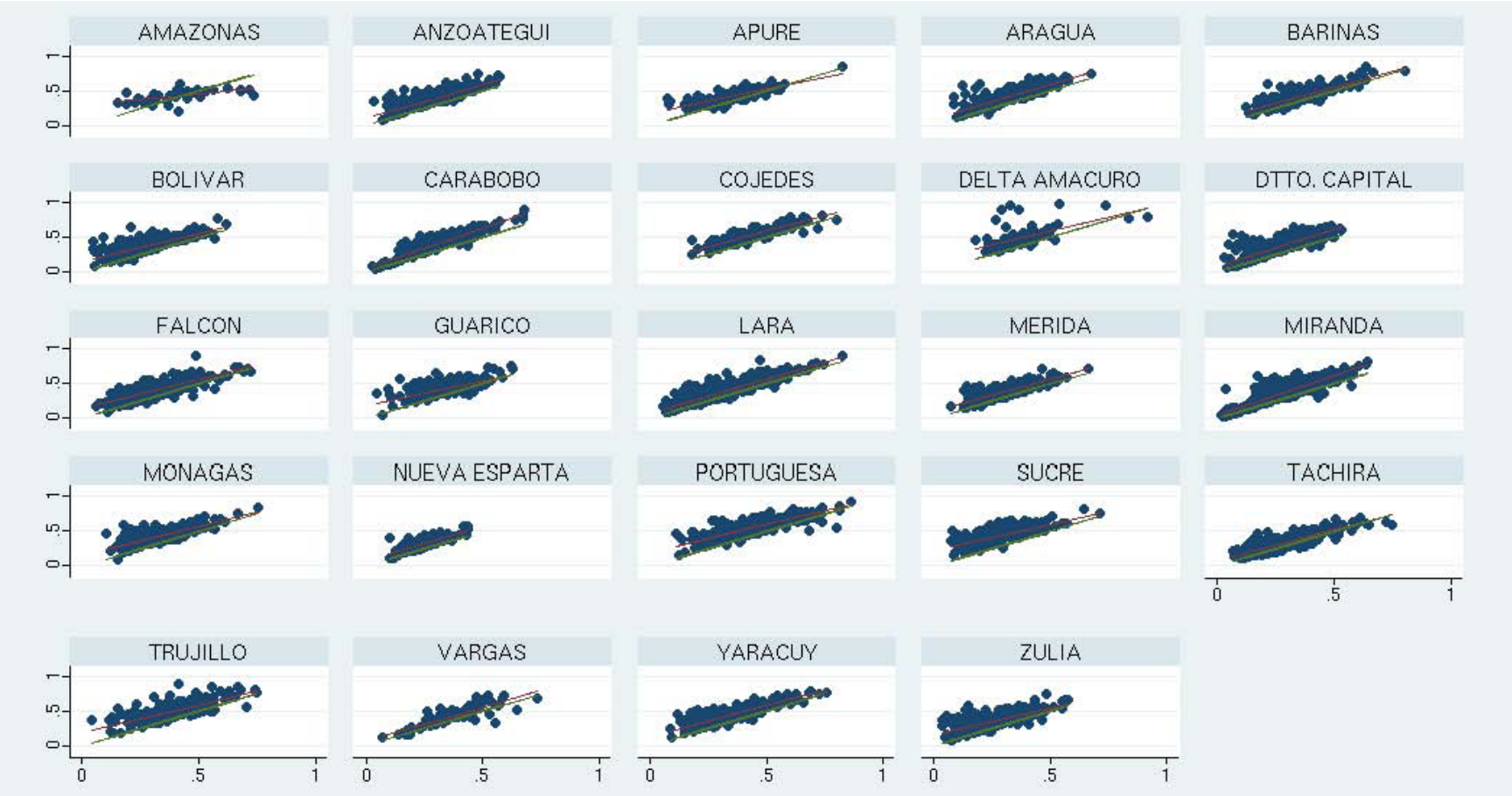


Figure 9: Relationship between *yes* shares in 2009 and *yes* shares in 2007. The red line represents the fitted results of a linear regression between the proportion of *yes* votes in the 2007 constitutional referendum (bundle A), and the proportion of *yes* votes in the 2009 constitutional referendum. The green line represents a situation where the proportion of *yes* votes in both elections is the same ($45°$ line).

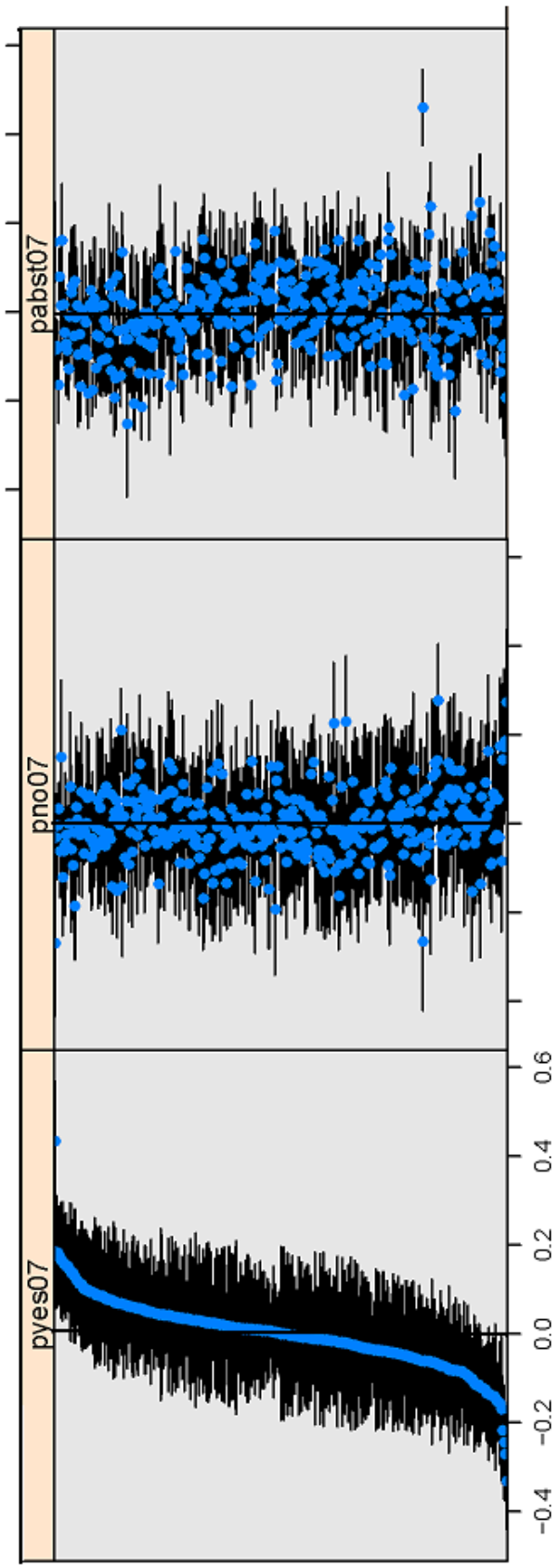
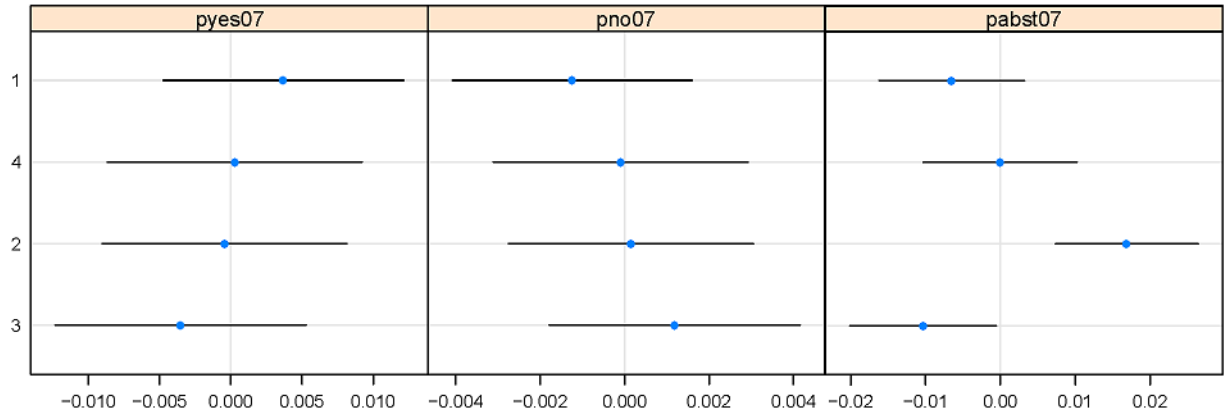
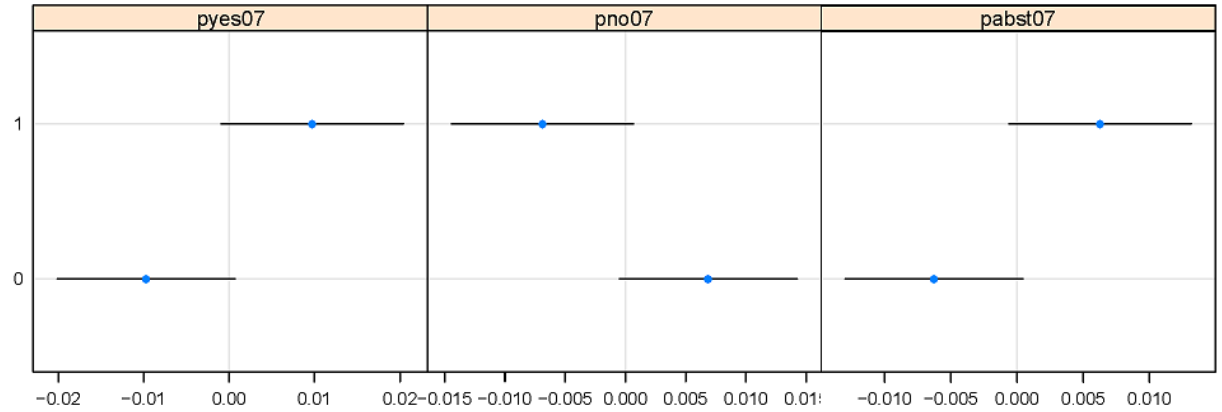


Figure 10: Goodman regression coefficients by region (deviations from the mean), with dependent variable equal to the yes proportion of the eligible.

a- Random coefficients by age quantiles



b- Random coefficients by gender[9]



c- Random coefficients by rurality quantiles

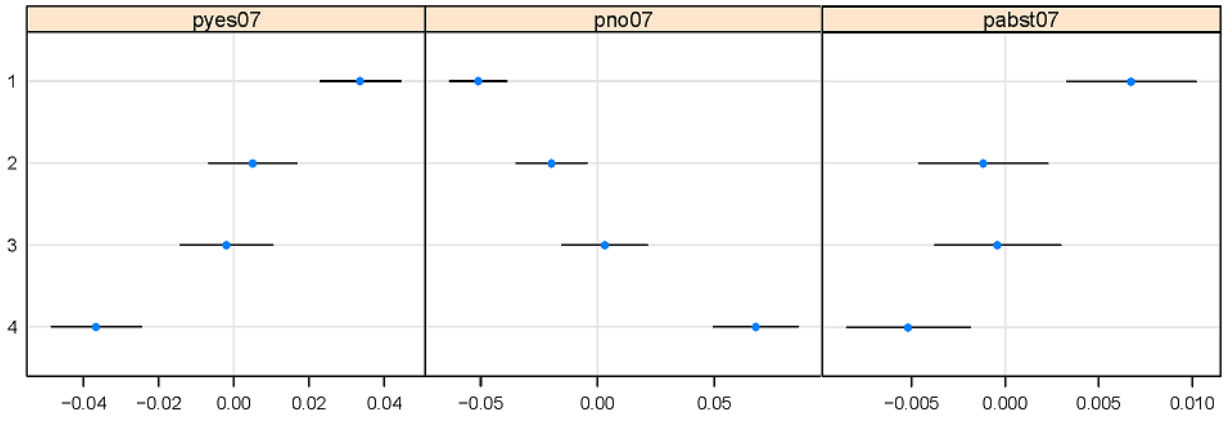


Figure 11: Goodman regression coefficients by demographic differences (deviations from the mean), with dependent variable equal to the yes proportion of the eligible. In the second panel, '1' indicates more than 50% female.

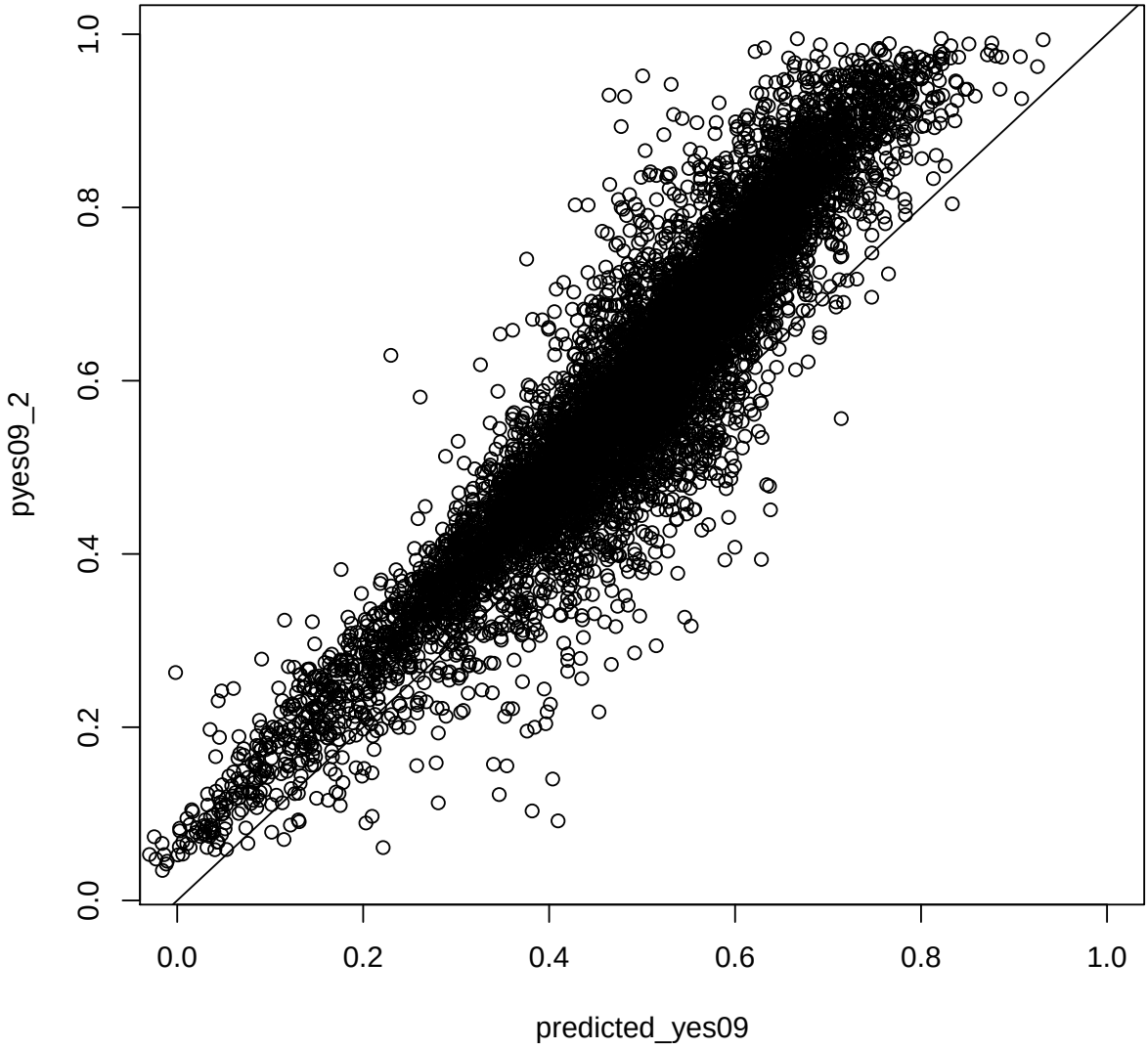


Figure 12: Forecasted 2009 results versus actual 2009 results. Dependent variable: yes votes as a proportion of total votes. Predictions were made based on the 2007 yes and no shares of the eligible electorate, using the estimated coefficients corresponding to an OLS regression where the dependent variable was the 2007 referendum yes share, and the independent variables were the 2006 Chavez and Rosales shares of the eligible electorate, plus a set of socio-demographic variables.